Key Concepts: Systems of two linear equations with two unknowns
Recognizing Inconsistent systems of equations
Recognizing dependent equations
Hot-diggity-dog word problems

Example 1

Use the method of substitution to find the solution to the system $\begin{cases} y = 2x - 9 \\ 4x - 2y = 1 \end{cases}$.

1. $y = 2x - 9$ is solved for y
2. substitute into $4x - 2y = 1$

$$4x - 2(2x - 9) = 1$$
$$4x - 4x + 18 = 1$$
$$18 = 1$$

Umm... I don't think so!

The system is inconsistent; it has no solutions!

The lines must be parallel!

$$4x - 2y = 1$$
$$4x - 1 = 2y$$
$$\frac{4x-1}{2} = \frac{2y}{2}$$
$$2x - \frac{1}{2} = y$$
$$m = 2$$

$$y = 2x - 9$$
$$m = 2$$

Parallel indeed!

Example 2

Use the addition method to find the solution to the system $\begin{cases} 9x - 6y = 21 \\ \frac{2y}{2} = \frac{3x}{2} - \frac{7}{2} \end{cases}$

$$\begin{cases} 9x - 6y = 21 \\ 2y = 3x - 7 \end{cases} \Rightarrow \begin{cases} 9x - 6y = 21 \\ -3x + 2y = -7 \end{cases}$$

$$\Rightarrow \begin{cases} 9x - 6y = 21 \\ 3(-3x + 2y) = 3(-7) \end{cases}$$

$$\Rightarrow \begin{cases} 9x - 6y = 21 \\ -9x + 6y = -21 \end{cases}$$

$0 = 0 \Leftarrow$ identity

Word!

Same freakin' line.

Every point on the line is a solution to the system. The are "infinitely many" solutions.

The equations are dependent.

Recognizing weird solutions when using the substitution or addition methods

- If, during the solution process, you *correctly* come up with an equation of the form $a = b$ where a and b are two *different* real numbers, then the system you are solving is inconsistent and has no solutions.
- If, during the solution process, you *correctly* come up with an equation of the form $a = a$ where a is a real number, then the equations in the system you are solving are dependent and the system has "infinitely many" solutions. In fact, the two equations represent the same line and any point on that line is a solution to the system of equations.

The solution set is $\{(x,y) \mid y = \frac{3}{2}x - \frac{7}{2}\}$

Example 3

Three Clifford Bars and six Zoned Bars contain a total of 1980 calories. Four Clifford Bars and One Zoned Bar contain a total of 1170 calories. How many calories are there in one of each type of bar?

Let x represent the number of calories in a Clifford bar and y the number of calories in a zoned bar.

$$\begin{cases} 3x + 6y = 1980 \\ 4x + y = 1170 \end{cases} \Longrightarrow \begin{cases} 3x + 6y = 1980 \\ -6(4x+y) = -6(1170) \end{cases}$$

$$\Longrightarrow \begin{cases} 3x + 6y = 1980 \\ -24x - 6y = -7020 \end{cases}$$

$$-21x = -5040$$

$$x = \frac{-5040}{-21} = 240$$

$$4(240) + y = 1170$$

$$y = 210$$

There are 240 calories in one Clifford bar + 210 calories in one Zoned bar.

Your Turn

One Clifford Bar and eight Zoned Bars contain a total of 530 *fat* calories. Six Clifford Bars and three Zoned Bar contain a total of 480 fat calories. How many fat calories are there in one of each type of bar?

Let x represent the number of fat calories in a Clifford bar and y the number of fat calories in a zoned bar.

$$\begin{cases} x + 8y = 530 \\ 6x + 3y = 480 \end{cases}$$

① Solve $x + 8y = 530$

$$x = -8y + 530$$

② Substitute into $6x + 3y = 480$

$$6(-8y + 530) + 3y = 480$$

$$-48y + 3180 + 3y = 480$$

$$-45y = -2700$$

$$y = \frac{-2700}{-45} = 60$$

③ backsub into $x = -8y + 530$

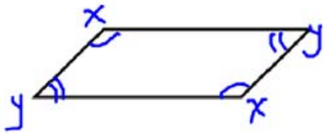
$$x = -8(60) + 530$$

$$= 50$$

There are 50 fat calories in one Clifford bar and 60 fat calories in one Zoned bar.

Example 4

A parallelogram has two pairs of congruent angles and the four angles always sum to 360°. For a certain parallelogram the larger angle is half the size of 40° more than three times the smaller angle. Find the two angles.



Let x represent the degree measure of the larger angles and y the degree measure of the smaller angles.

$$\begin{cases} 2x + 2y = 360^\circ \\ x = \dfrac{3y + 40^\circ}{2} \end{cases}$$

Substitute into $2x + 2y = 360^\circ$

$$\cancel{2}\left(\frac{3y + 40^\circ}{\cancel{2}}\right) + 2y = 360^\circ$$

$$3y + 40^\circ + 2y = 360^\circ$$

$$5y = 320^\circ$$

$$y = 64^\circ$$

$$x = \frac{3(64^\circ) + 40^\circ}{2} = 116^\circ$$

∴ The angle measures are 64° + 116°.

Your Turn

The degree measurements of the angles of any triangle add up to 180°. In an isosceles triangle, two of the angles have equal measure. For a certain isosceles triangle the two congruent angles each have measurement that is 10° less than twice the measure of the non-congruent angle. Find the measurement of each of the angles.

Let x be the degree measure of the congruent angles and y the degree measure of the third angle

$$\begin{cases} 2x + y = 180^\circ \\ x = 2y - 10^\circ \end{cases}$$

substitute into $2x + y = 180^\circ$

$$2(2y - 10^\circ) + y = 180^\circ$$

$$4y - 20^\circ + y = 180^\circ$$

$$5y = 200^\circ$$

$$y = 40^\circ$$

$$x = 2(40^\circ) - 10^\circ = 70^\circ$$

The congruent angles each measure 70° & the third angle measures 41°

Example 5

Goofus has gone and done it again - he was playing ball in the house and accidentally broke his mom's favorite vase. He needs to buy a replacement before his mom gets home, but Goofus spent all of his money on downloads. Goofus calls his good pal Gallant who agrees to lend Goofus the money if Goofus will pay back the loan plus a 15% loan fee. Goofus already owes Gallant on a previous loan where Gallant only charged a 10% loan fee. After agreeing to the new loan, Goofus now owes Gallant a total of $24.30 in loan fees. The total amount that Goofus borrowed was $213. What was the original amount of each of the loans (*before* the loan fees).

	amount borrowed, principal P	interest rate r	amount of interest $I = Pr$
first loan	x	$.10$	$.10x$
Second loan	y	$.15$	$.15y$

Let x represent the first amount borrowed ($) and y the second amount borrowed ($)

$$\begin{cases} \text{total borrowed} = \$213 \\ \text{total interest} = \$24.30 \end{cases} \Rightarrow \begin{cases} x+y = \$213 \\ .10x+.15y = \$24.30 \end{cases}$$

$$\Rightarrow \begin{cases} -.10(x+y) = -.10(\$213) \\ .10x+.15y = \$24.30 \end{cases}$$

$$\Rightarrow \begin{cases} -.10x - .10y = -\$21.30 \\ .10x + .15y = \$24.30 \end{cases}$$

$$.05y = \$3.00$$

$$y = \frac{\$3.00}{.05} = \$60$$

$x+y = \$213$

$x + \$60 = \$213$

$x = \$153$

The first loan was for $153 and the second was for $60.

check interest

$.10(\$153) + .15(\$60) = \$15.30 + \$9.00$

$= \$24.30$ ✓

Your Turn

Polly Wannabe invested a total of $19,000 in two accounts. One account earned 5% simple interest over the following year and the other account earned 7.25% simple interest over that year. Ms. Wannabe earned a total of $1211 in interest over the year. How much did Ms. Wannabe initially invest in each account?

	P	r	I
first account	x	.05	$.05x$
Second account	y	.0725	$.0725y$

Let x represent the amount invested at 5%
and y the amount invested at 7.25%

$$\begin{cases} x+y = \$19000 \\ .05x+.0725y = \$1211 \end{cases} \Rightarrow \begin{cases} -.05(x+y) = -.05(\$19000) \\ .05x+.0725y = \$1211 \end{cases}$$

$$\Rightarrow \begin{cases} -.05x-.05y = -\$950 \\ .05x+.0725y = \$1211 \end{cases}$$

$$.0225y = \$261$$

$$y = \frac{\$261}{.0225} = \$11,600$$

$$x+y = \$19000$$

$$x+\$11,600 = \$19000$$

$$x = \$7,400$$

Polly Wannabe invested \$7,400 @ 5% interest and \$11,600 @ 7.25% interest.

Example 6

Aye Carumba! Bonita's chemistry instructor has told her to put 2 liters of a solution that is 30% acid into a beaker. But her silly instructor has given her one bottle of solution that is 15% acid and another bottle of solution that is 35% acid. There's no solution that is 30% acid in the entire, friggin' room. As usual, Bonita's lab partner, Stan, is no help at all. Bonita really wants to get into the nursing program, so can you please give her some help in resolving this dilemma?

	Amount of Solution → T	% acid r	amount of acid A = rT
Weak acid	x	.15	.15x
Strong acid	y	.35	.35y

Let x represent the amount of weak acid used (ℓ) and y the amount of strong acid used (ℓ)

$$\begin{cases} \text{total amount of solution} = 2\ell \\ \text{total amount of acid} = 30\% \text{ of } 2\ell \end{cases}$$

$$\Rightarrow \begin{cases} x + y = 2\ell \\ .15x + .35y = .30(2\ell) \end{cases}$$

$$\Rightarrow \begin{cases} x + y = 2\ell \\ .15x + .35y = .6\ell \end{cases} \Rightarrow \begin{cases} -.15(x+y) = -.15(2\ell) \\ .15x + .35y = .6\ell \end{cases}$$

$$\Rightarrow \begin{cases} -.15x - .15y = -.3\ell \\ .15x + .35y = .6\ell \end{cases}$$

$$.2y = .3\ell$$

$$y = \frac{.3\ell}{.2} = 1.5\ell$$

$$x + 1.5\ell = 2\ell \Rightarrow x = .5\ell$$

Bonita needs to use 1.5 ℓ of the strong acid and ½ ℓ of the weak acid.

Check Acid Amount

$$.15(.5\ell) + .35(1.5\ell) = .6\ell \checkmark$$

Your Turn

Juniper Bean runs a coffee stand. Juniper makes a blend using two different types of bean. The Belizean beans sell for \$8.96 per pound and the Kenyan Beans sell for \$4.48 per pound. Juniper made 32 pounds of the blend and the blend was sold at \$7.28 per pound. How many pounds of each type of bean did Juniper use in the blend? (Assume that Juniper made the same amount on the blend as she would have made selling the beans separately.)

Hint: Your variables are the numbers of pounds of each type of bean used. One of your equations is about the total weight of the beans. The other equation is about the total selling price of the blend (i.e., how much would it cost if someone bought all 32 pounds of the bean and where did this price come from?)

Let x represent the weight of Belizean bean used (lb) and y the weight of Kenyan Beans used (lb).

$$\begin{cases} \text{total weight} = 32 \text{ lb} \\ \text{total value} = \left(7.28 \frac{\$}{\text{lb}}\right)(32 \text{ lb}) \end{cases}$$

$$\Rightarrow \begin{cases} x + y = 32 \\ 8.96x + 4.48y = 232.96 \end{cases}$$

$$\Rightarrow \begin{cases} -4.48(x+y) = -4.48(32) \\ 8.96x + 4.48y = 232.96 \end{cases}$$

$$\Rightarrow \begin{cases} -4.48x - 4.48y = -143.36 \\ 8.96x + 4.48y = 232.96 \end{cases}$$

$$4.48x = 89.6$$

$$x = \frac{89.6}{4.48} = 20$$

$$20 + y = 32 \Rightarrow y = 12$$

Ms. Bean used 20 lb of the Belizean beans and 12 lbs of the Kenyan beans.