

Key Concepts: Systems of two linear equations with two unknowns
 The substitution method
 The elimination method (AKA the addition method)

The substitution strategy

1. Solve, if necessary, one equation for one of the variables.
2. Substitute the resultant expression for the variable you solved for into the other equation.
3. Solve the resultant linear equation in one variable.
4. Use the value found in step (3) and the equation from step (1) to find the value of the other variable.
5. Check your solution in *both original equations*.
6. When your solution checks, state your solution using a complete sentence.

Example 1

Use the method of substitution to find the solution to the system $\begin{cases} 3x - 2y = -17 \\ x - 4y = 1 \end{cases}$.

1. Solve $x - 4y = 1$ for x

$$\begin{aligned} x - 4y &= 1 \\ x - 4y + 4y &= 1 + 4y \\ x &= 1 + 4y \end{aligned}$$

2. Substitute into $3x - 2y = -17$

$$\begin{aligned} 3(1 + 4y) - 2y &= -17 \\ 3 + 12y - 2y &= -17 \\ 10y + 3 &= -17 \\ 10y + 3 - 3 &= -17 - 3 \\ 10y &= -20 \\ \frac{10y}{10} &= \frac{-20}{10} \\ y &= -2 \end{aligned}$$

3. Back sub

into $x = 1 + 4y$

$$\begin{aligned} x &= 1 + 4(-2) \\ x &= -7 \end{aligned}$$

4. check $(-7, -2)$

$$3x - 2y = -17$$

$$3(-7) - 2(-2) \stackrel{?}{=} -17$$

$$-21 + 4 = -17 \checkmark$$

$$x - 4y = 1$$

$$\begin{aligned} -7 - 4(-2) &= 1 \\ -7 + 8 &= 1 \checkmark \end{aligned}$$

The solution to the system is $(-7, -2)$.

Example 2

Use the method of substitution to find the solution to the system $\begin{cases} -2x - y = 21 \\ y = 4x + 9 \end{cases}$.

1. $y = 4x + 9$ is solved for y

2. substitute into $-2x - y = 21$

$$-2x - (4x + 9) = 21$$

$$-2x - 4x - 9 = 21$$

$$-6x = 30$$

3. Backsub into $y = 4x + 9$ $x = -5$

$$y = 4(-5) + 9$$

$$y = -11$$

4. check $(-5, -11)$

$$\begin{array}{r} -2x - y = 21 \\ -2(-5) - (-11) = 21 \\ 10 + 11 = 21 \checkmark \end{array}$$

$$\begin{array}{r} y = 4x + 9 \\ -11 = 4(-5) + 9 \\ -11 = -20 + 9 \checkmark \end{array}$$

The solution to the system is $(-5, -11)$.

Example 3

Use the method of substitution to find the solution to the system $\begin{cases} 6x - 5y = 9 \\ 30x + 10y = -4 \end{cases}$

1. Solve $30x + 10y = -4$ for y

$$30x + 10y = -4$$

$$10y = -4 - 30x$$

$$\frac{10y}{10} = \frac{-4 - 30x}{10}$$

$$y = \frac{-4}{10} - \frac{30x}{10}$$

$$y = -\frac{2}{5} - 3x$$

2. Substitute into $6x - 5y = 9$.

$$6x - 5\left(-\frac{2}{5} - 3x\right) = 9$$

$$6x + 2 + 15x = 9$$

$$21x = 7$$

$$x = \frac{7}{21} = \frac{1}{3}$$

3. Backsub into $y = -\frac{2}{5} - 3x$

$$y = -\frac{2}{5} - 3\left(\frac{1}{3}\right)$$

$$y = -\frac{2}{5} - 1$$

$$y = -\frac{7}{5}$$

4. check $\left(\frac{1}{3}, -\frac{7}{5}\right)$

$$6x - 5y = 9$$

$$6\left(\frac{1}{3}\right) - 5\left(-\frac{7}{5}\right) \stackrel{?}{=} 9$$

$$2 + 7 = 9$$

$$30x + 10y = -4$$

$$30\left(\frac{1}{3}\right) + 10\left(-\frac{7}{5}\right) \stackrel{?}{=} -4$$

$$10 - 14 = -4$$

The solution to the system is $\left(\frac{1}{3}, -\frac{7}{5}\right)$.

The addition strategy (AKA the elimination strategy)

1. Write both equations in standard form.
2. Multiply both sides of one equation by a number so that when you add the respective sides of the two equations together, one of the variables is eliminated. Sometimes this objective can be more easily achieved by multiplying both sides of one equation by one number and both sides of the other equation by a different number before adding the respective sides of the two equations.
3. Solve the resultant linear equation in one variable.
4. Use the value found in step (3) and one of the original equations to find the value of the other variable.
5. Check your solution in *both original equations*.
6. When your solution checks, state your solution using a complete sentence.

Example 4

Use the elimination method to find the solution to the system $\begin{cases} 3x - y = 22 \\ 4x + 5y = -34 \end{cases}$

$$\begin{cases} 3x - y = 22 \\ 4x + 5y = -34 \end{cases} \Rightarrow \begin{cases} 5(3x - y) = 5(22) \\ 4x + 5y = -34 \end{cases}$$

$$\Rightarrow \begin{cases} 15x - 5y = 110 \\ 4x + 5y = -34 \end{cases}$$

$$19x = 76$$

$$x = \frac{76}{19} = 4$$

Backsub into $3x - y = 22$

$$3(4) - y = 22$$

$$-10 = y$$

Check! (4, -10)

$$3x - y = 22$$

$$3(4) - (-10) = 22?$$

$$12 + 10 = 22 \checkmark$$

$$4x + 5y = -34$$

$$4(4) + 5(-10) = -34?$$

$$16 - 50 = -34 \checkmark$$

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The solution is (4, -10).

Example 5

Use the addition method to find the solution to the system $\begin{cases} -x + 4y = 15 \\ -x - 3y = 1 \end{cases}$.

$$\begin{cases} -x + 4y = 15 \\ -x - 3y = 1 \end{cases} \Rightarrow \begin{cases} 3(-x + 4y) = 3(15) \\ 4(-x - 3y) = 4(1) \end{cases}$$

$$\Rightarrow \begin{array}{r} \begin{cases} -3x + 12y = 45 \\ -4x - 12y = 4 \end{cases} \\ \hline -7x \qquad = 49 \\ x \qquad = -7 \end{array}$$

Back sub into $-x + 4y = 15$

$$\begin{aligned} -(-7) + 4y &= 15 \\ 7 + 4y &= 15 \\ 4y &= 8 \\ y &= 2 \end{aligned}$$

Check $(-7, 2)$

$$\begin{array}{ll} -x + 4y = 15 & -x - 3y = 1 \\ -(-7) + 4(2) \stackrel{?}{=} 15 & -(-7) - 3(2) \stackrel{?}{=} 1 \\ \quad \quad \quad 7 - 6 = 1 & \quad \quad \quad 7 - 6 = 1 \end{array}$$

The solution to the system is $(-7, 2)$.

Example 6

Use the addition method to find the solution to the system $\begin{cases} 3x = -4y + 4 \\ 5y = 2x + 5 \end{cases}$.

$$\begin{aligned} \begin{cases} 3x &= -4y + 4 \\ 5y &= 2x + 5 \end{cases} &\Rightarrow \begin{cases} 3x + 4y &= 4 \\ -2x + 5y &= 5 \end{cases} \\ &\Rightarrow \begin{cases} 2(3x + 4y) &= 2(4) \\ 3(-2x + 5y) &= 3(5) \end{cases} \\ &\Rightarrow \begin{cases} 6x + 8y &= 8 \\ -6x + 15y &= 15 \end{cases} \\ &\quad \underline{\hspace{10em}} \\ &\quad \quad 23y = 23 \\ &\quad \quad y = 1 \end{aligned}$$

Backsub into $3x = -4y + 4$

$$3x = -4(1) + 4$$

$$3x = 0$$

$$\frac{3x}{3} = \frac{0}{3}$$

$$x = 0$$

Check $(0, 1)$

$$3(0) = -4(1) + 4$$

$$5(1) = 2(0) + 5$$

The solution
is $(0, 1)$.

$$3 \cdot \frac{x}{2} - \frac{4y}{3} = -\frac{13}{2}$$

$$\frac{3x}{2} - \frac{4y}{3} = -\frac{13}{2}$$

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Example 7

Use the elimination method to find the solution to the system

$$\begin{cases} \frac{x}{2} - \frac{4y}{3} = -\frac{13}{2} \\ \frac{3x}{4} + 2y = \frac{9}{4} \end{cases}$$



$$\begin{cases} \frac{x}{2} - \frac{4y}{3} = -\frac{13}{2} \\ \frac{3x}{4} + 2y = \frac{9}{4} \end{cases} \Rightarrow \begin{cases} 6\left(\frac{x}{2} - \frac{4y}{3}\right) = 6\left(-\frac{13}{2}\right) \\ 4\left(\frac{3x}{4} + 2y\right) = 4\left(\frac{9}{4}\right) \end{cases}$$

$$\Rightarrow \begin{cases} 3x - 8y = -39 \\ 3x + 8y = 9 \end{cases} \quad \begin{matrix} \text{Lucky} \\ \text{us!} \end{matrix}$$

$$\hline 6x = -30$$

$$x = -5$$

Backsub into $3x + 8y = 9$

$$3(-5) + 8y = 9$$

$$8y = 24$$

$$y = 3$$

Check $(-5, 3)$

$$\frac{x}{2} - \frac{4y}{3} = -\frac{13}{2}$$

$$-\frac{5}{2} - \frac{4(3)}{3} \stackrel{?}{=} -\frac{13}{2}$$

$$-\frac{5}{2} - 4 \stackrel{?}{=} -\frac{13}{2}$$

$$-\frac{5}{2} - 4 = -\frac{13}{2}$$

$$\frac{3x}{4} + 2y = \frac{9}{4}$$

$$\frac{3(-5)}{4} + 2(3) \stackrel{?}{=} \frac{9}{4}$$

$$-\frac{15}{4} + \frac{24}{4} = \frac{9}{4} \checkmark$$

The solution is $(-5, 3)$.

Please work each of the following problems on your own paper. That is, do not work these on this sheet of paper.

1. Solve, using the method of substitution, the system $\begin{cases} -x + 4y = 19 \\ x = -2y + 5 \end{cases}$.

2. Solve, using the method of substitution, the system $\begin{cases} 2x - 3y = 7 \\ 3x + 6y = 0 \end{cases}$.

3. Solve, using the addition method, the system $\begin{cases} 2x - 3y = 19 \\ 3x + 6y = -3 \end{cases}$.

4. Solve, using the elimination method, the system $\begin{cases} \frac{x}{3} + \frac{2y}{3} = 2 \\ -2x + \frac{y}{2} = 15 \end{cases}$.

5. Solve, using the elimination method, the system $\begin{cases} y = 5x + 82 \\ x = -2y + 54 \end{cases}$.

