Key Concepts: Systems of two linear equations with two unknowns
Solving systems graphically
Types of solution sets to systems of linear equations

Definition

A solution to a system of two linear equations with two unknowns is an ordered pair that makes each of the equations true.

Examples

Decide whether or not the given ordered pairs are solutions to the system of equations:

$$\begin{cases} 2x+3y=7 \\ 3x-2y=-9 \end{cases}$$

Is $(5,-1)$ a solution to the system?

$2(5)+3(-1) \stackrel{?}{=} 7$
$10-3=7$ ✓

$3(5)-2(-1) \stackrel{?}{=} -9$
$15+2 \neq -9$

$(5,-1)$ is <u>not</u> a solution to this system.

Is $(-1,3)$ a solution to the system?

$2(-1)+3(3) \stackrel{?}{=} 7$
$-2+9=7$ ✓

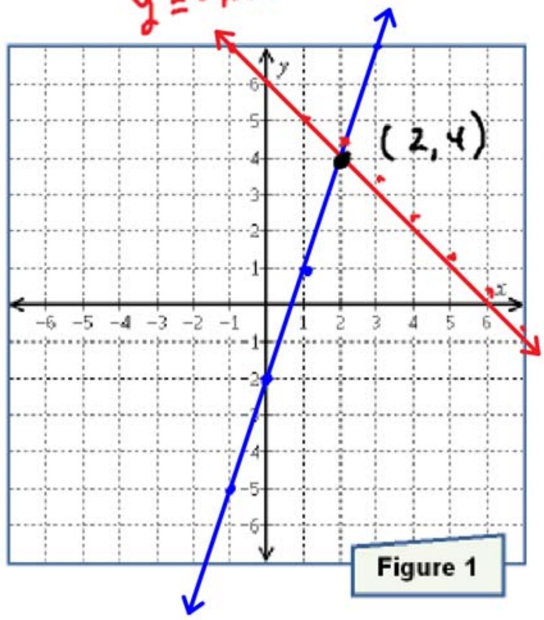
$3(-1)-2(3) = -9$?
$-3-6=-9$ ✓

$(-1,3)$ <u>is</u> a solution to the system.

Solve each system of equations by graphing the two linear equations and finding the point(s) they have in common. Make sure that you check your solution before stating your conclusion.

$y = 3x - 2$

Solve the system $\begin{cases} y = 3x - 2 \\ y = -x + 6 \end{cases}$.



$y = 3x - 2$

Slope is 3

y-intercept: $(0, -2)$

$y = -x + 6$

$m = -1$

y-intercept: $(0, 6)$

Check $(2,4)$

$4 \stackrel{?}{=} 3(2) - 2$ $4 \stackrel{?}{=} -2 + 6$ ✓

$4 = 6 - 2$ ✓ $y = -x + 6$

$y = 3x - 2$

The solution to the system is $(2, 4)$.

$$\frac{x + 2y = -3}{y = -1}$$

$$x - 2 = -3$$

$$x = -1$$

Solve the system $\begin{cases} y = \dfrac{x}{4} \\ x + 2y = -3 \end{cases}$.

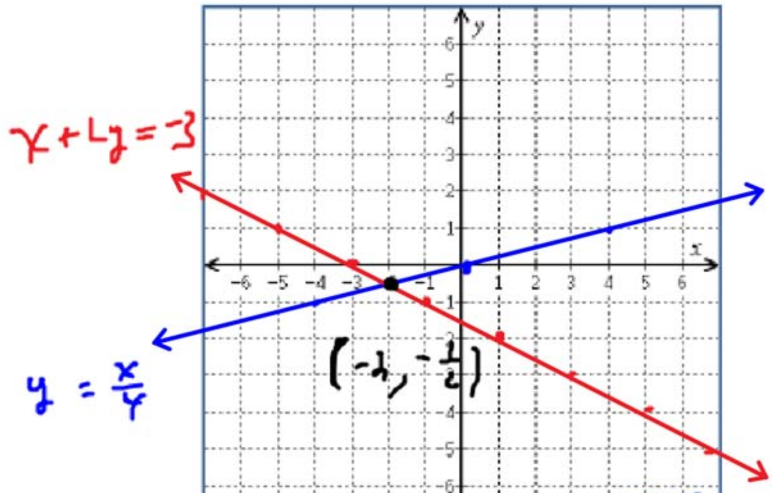


Figure 2

$$\frac{y = \frac{x}{4}}{}$$

$$y = \frac{1}{4}x$$

$$m = \frac{1}{4}$$

y-int: $(0,0)$

$$\frac{x+2y=-3}{}$$

$$x + 2y = -3$$

$$x + 2y - x = -3 - x$$

$$2y = -x - 3$$

$$\frac{2y}{2} = \frac{-x-3}{2}$$

$$y = -\frac{x}{2} - \frac{3}{2}$$

$$y = -\frac{1}{2}x - \frac{3}{2}$$

$$m = -\frac{1}{2}$$

y-intercept: $(0, -\frac{3}{2})$ ⇐ stinky pt. to graph

Let's find a non-stinky point: $(-5, 1)$

Check $(-2, -\frac{1}{2})$

$$\frac{y = \frac{x}{4}}{-\frac{1}{2} = \frac{-2}{4}} \checkmark$$

$$\frac{x + 2y = -3}{-2 + 2(-\frac{1}{2}) = -3}$$

$$-2 - 1 = -3 \checkmark$$

The solution to the system is $(-2, -\frac{1}{2})$.

Solve the system $\begin{cases} 2x - y = 9 \\ x = 3 \end{cases}$.

$2x - y = 9$
$x = 3$
$2(3) - y = 9$
$6 - y = 9$
$-y = 3$
$y = -3$

$2x - y = 9$
$2x = y + 9$
$2x - 9 = y$
$y = 2x - 9$

$m = 2$
y-intercept: $(0, -9)$ ← starting pt.
$(1, -7)$ ← goal pt.

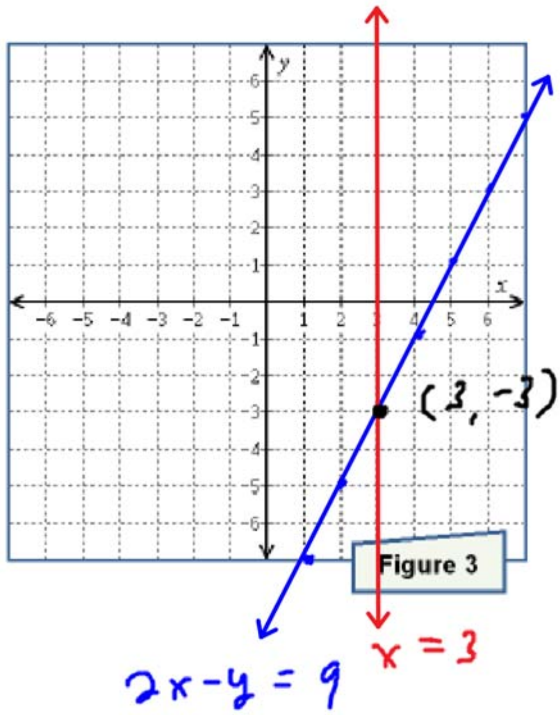


Figure 3

Check $(3, -3)$

$2x - y = 9$?
$2(3) - (-3) = 9$?
$6 + 3 = 9$ ✓

$x = 3$
$3 = 3$? umm...

The solution to the system is $(3, -3)$

Solve the system $\begin{cases} 4x + 6y = 24 \\ y = -\frac{2}{3}x - 1 \end{cases}$

$4x + 6y = 24$

$6y = -4x + 24$

$\frac{6y}{6} = \frac{-4x + 24}{6}$

$y = -\frac{2}{3}x + 4$

$4x + 6y = 24$

$y = -\frac{2}{3}x + 1$

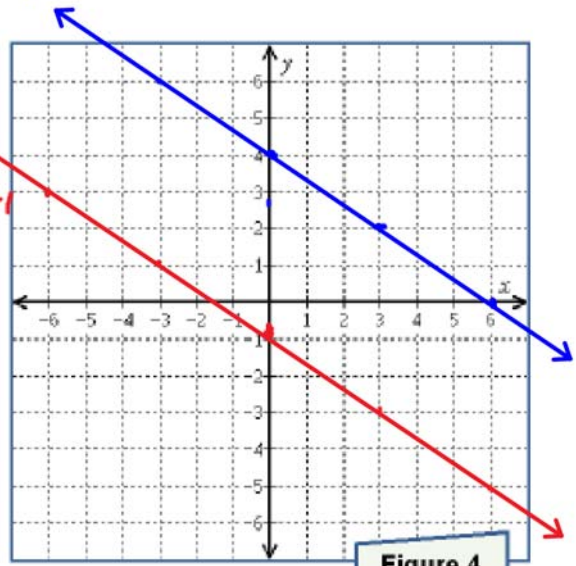


Figure 4

$y = -\frac{2}{3}x - 1$

$m = -\frac{2}{3}$

y-intercept: $(0, -1)$

Parallel lines!

The system has no solutions.

Solve the system $\begin{cases} y = 3x - 2 \\ 6x - 2y = 4 \end{cases}$.

$y = 3x - 2$

$m = 3$

y-intercept: $(0, -2)$

$6x - 2y = 4$

$6x - 4 = 2y$

$3x - 2 = y$

Same frickin line!

$y = 3x - 2$

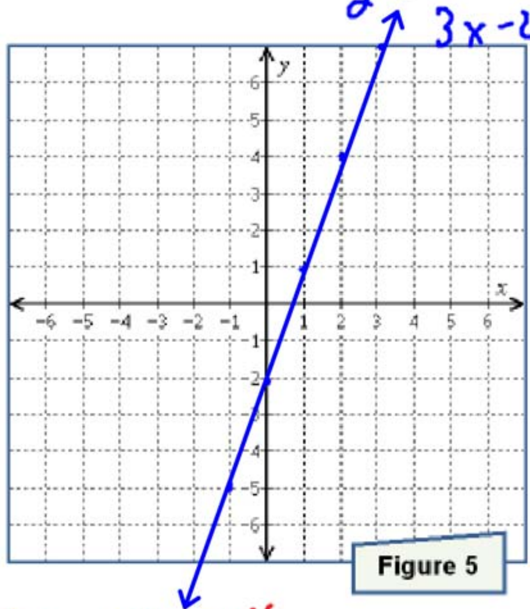


Figure 5

$6x - 2y = 4$

The system has an unlimited number of solutions.

Types of solutions sets to systems of two linear equations with two unknowns

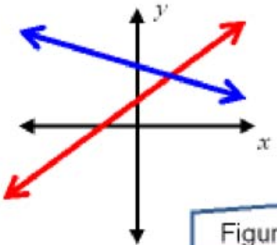


Figure 6

A system can have exactly one solution. In this case, the system is called ***consistent*** and the equations are called ***independent***. This happens when the two equations graph to lines that intersect at a single point.

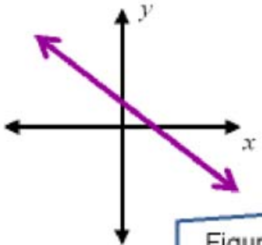


Figure 7

A system can have an "infinite number" of solutions. In this case, the system is called ***consistent*** and the equations are called ***dependent***. This happens when the two equations graph to the same line

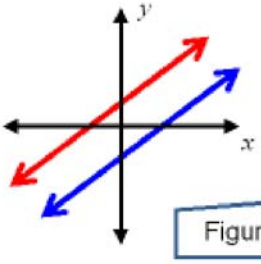


Figure 8

A system can have no solution. In this case, the system is called ***inconsistent*** and the equations are called ***independent***. This happens when the two equations graph to parallel lines.

Examples

For each system, write both equations in *slope-intercept* form and decide - without graphing - whether the system is consistent or inconsistent and whether the equations are dependent or independent.

$$\begin{cases} y = 2x - 2 \\ 4x = 2y - 4 \end{cases}$$

$y = 2x - 2$

$4x = 2y - 4$

$4x + 4 = 2y$

$2x + 2 = y$

Parallel lines / no solution!

The system is inconsistent.

The equations are independent.

$$\begin{cases} 2x - 3y = 8 \\ 3x - 2y = 8 \end{cases}$$

$2x - 3y = 8$

$2x = 3y + 8$

$2x - 8 = 3y$

$\frac{2}{3}x - \frac{8}{3} = y$

$3x - 2y = 8$

$3x = 2y + 8$

$3x - 8 = 2y$

$\frac{3}{2}x - 4 = y$

different slopes / the lines intersect!

The system is consistent.

The equations are independent.

Find the solution to the system of equations $\begin{cases} y = \dfrac{x}{3} \\ 2x + y = -7 \end{cases}$.

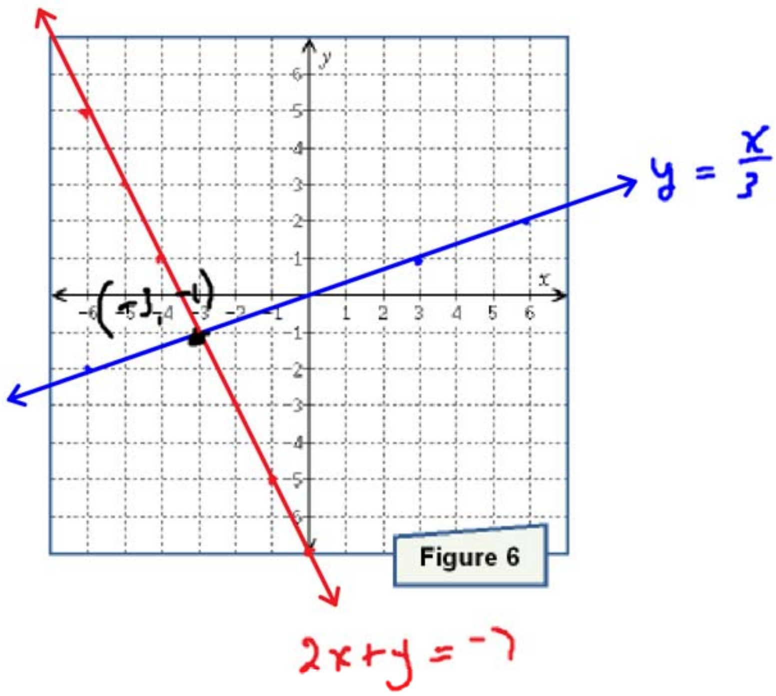


Figure 6

$2x + y = -7$

$y = -2x - 7$

Check $(-3, -1)$

$$\frac{y = \frac{x}{3}}{-1 = -\frac{3}{3}} \checkmark$$

$$2x + y \stackrel{?}{=} -7$$

$$2(-3) + (-1) = -7 \checkmark$$

The solution is $(-3, -1)$.