

Problem Set 1

$$\begin{aligned} \text{a. } x^2 + 12x + 32 &= x^2 + 4x + 8x + 32 \\ &= x(x + 4) + 8(x + 4) \\ &= (x + 8)(x + 4) \end{aligned}$$

$$\begin{aligned} \text{b. } y^2 + 38y + 72 &= y^2 + 2y + 36y + 72 \\ &= y(y + 2) + 36(y + 2) \\ &= (y + 36)(y + 2) \end{aligned}$$

$$\begin{aligned} \text{c. } x^2 + 18x + 72 &= x^2 + 6x + 12x + 72 \\ &= x(x + 6) + 12(x + 6) \\ &= (x + 12)(x + 6) \end{aligned}$$

$$\text{d. } x^2 + 30x + 72 \text{ is prime}$$

$$\begin{aligned} \text{e. } a^2 + 14a + 49 &= (a + 7)(a + 7) \\ &= (a + 7)^2 \end{aligned}$$

$$\text{f. } y^2 + 91y + 90 = (y + 90)(y + 1)$$

$$\text{g. } t^2 + 29t + 180 = (t + 9)(t + 20)$$

$$\text{h. } a^2 + 4a + 24 \text{ is prime}$$

Problem Set 2

$$\begin{aligned} \text{a. } x^2 - 14x + 48 &= x^2 - 6x - 8x + 48 \\ &= x(x - 6) - 8(x - 6) \\ &= (x - 8)(x - 6) \end{aligned}$$

$$\begin{aligned} \text{b. } b^2 - 27b + 92 &= b^2 - 4b - 23b + 92 \\ &= b(b - 4) - 23(b - 4) \\ &= (b - 23)(b - 4) \end{aligned}$$

$$\text{c. } x^2 - 29x + 92 \text{ is prime}$$

$$\text{d. } y^2 - 26y + 48 = (y - 24)(y - 2)$$

$$\text{e. } x^2 - 16x + 60 = (x - 10)(x - 6)$$

$$\text{f. } a^2 - 30a + 81 = (a - 27)(a - 3)$$

$$\text{g. } t^2 - 27t + 27 \text{ is prime}$$

$$\begin{aligned} \text{h. } m^2 - 30m + 225 &= (m - 15)(m - 15) \\ &= (m - 15)^2 \end{aligned}$$

Problem Set 3

$$\begin{aligned} \text{a. } t^2 + 9t - 90 &= t^2 - 6t + 15t - 90 \\ &= t(t - 6) + 15(t - 6) \\ &= (t + 15)(t - 6) \end{aligned}$$

$$\begin{aligned} \text{b. } t^2 + 5t - 24 &= t^2 - 3t + 8t - 24 \\ &= t(t - 3) + 8(t - 3) \\ &= (t + 8)(t - 3) \end{aligned}$$

$$\begin{aligned} \text{c. } z^2 + 15z - 100 &= z^2 - 5z + 20z - 100 \\ &= z(z - 5) + 20(z - 5) \\ &= (z + 20)(z - 5) \end{aligned}$$

$$\text{d. } y^2 + y - 90 = (y + 10)(y - 9)$$

e. $m^2 + 20m - 44 = (m + 22)(m - 2)$

f. $p^2 + 200p - 100$ is prime

g. $x^2 + 4x - 24$ is prime

h. $w^2 + 22w - 240 = (w - 8)(w + 30)$

Problem Set 4

a.
$$\begin{aligned} x^2 - 6x - 16 &= x^2 + 2x - 8x - 16 \\ &= x(x + 2) - 8(x + 2) \\ &= (x - 8)(x + 2) \end{aligned}$$

b.
$$\begin{aligned} t^2 - 16t - 80 &= t^2 + 4t - 20t - 80 \\ &= t(t + 4) - 20(t + 4) \\ &= (t - 20)(t + 4) \end{aligned}$$

c.
$$\begin{aligned} p^2 - 11p - 80 &= p^2 + 5p - 16p - 80 \\ &= p(p + 5) - 16(p + 5) \\ &= (p - 16)(p + 5) \end{aligned}$$

d. $y^2 - 2y - 48 = (y - 8)(y + 6)$

e. $x^2 - 20x - 48$ is prime

f. $w^2 - 3w - 180 = (w - 15)(w + 12)$

g. $z^2 - 88z - 180 = (z - 90)(z + 2)$

h. $w^2 - 399w - 400 = (w - 400)(w + 1)$

Problem Set 5

a.
$$\begin{aligned} 10x^2 + 39x + 14 &= 10x^2 + 4x + 35x + 14 \\ &= 2x(5x + 2) + 7(5x + 2) \\ &= (2x + 7)(5x + 2) \end{aligned}$$

b.
$$\begin{aligned} 14w^2 - 31w - 10 &= 14w^2 + 4w - 35w - 10 \\ &= 2w(7w + 2) - 5(7w + 2) \\ &= (2w - 5)(7w + 2) \end{aligned}$$

c.
$$\begin{aligned} 12y^2 - 19y + 4 &= 12y^2 - 3y - 16y + 4 \\ &= 3y(4y - 1) - 4(4y - 1) \\ &= (3y - 4)(4y - 1) \end{aligned}$$

d.
$$\begin{aligned} 6y^2 + 47y - 8 &= 6y^2 - y + 48y - 48 \\ &= y(6y - 1) + 48(y - 1) \\ &= (y + 48)(6y - 1) \end{aligned}$$

e.
$$\begin{aligned} 6b^2 + 13b - 8 &= 6b^2 - 3b + 16b - 8 \\ &= 3b(2b - 1) + 8(2b - 1) \\ &= (3b + 8)(2b - 1) \end{aligned}$$

f.
$$\begin{aligned} 4t^2 - 17t + 4 &= 4t^2 - 16t - t + 4 \\ &= 4t(t - 4) - 1(t - 4) \\ &= (4t - 1)(t - 4) \end{aligned}$$

g.
$$\begin{aligned} 4x^2 + 12x + 9 &= 4x^2 + 6x + 6x + 9 \\ &= 2x(2x + 3) + 3(2x + 3) \\ &= (2x + 3)(2x + 3) \\ &= (2x + 3)^2 \end{aligned}$$

h.
$$\begin{aligned} 8w^2 + 7w - 1 &= 8w^2 + 8w - w - 1 \\ &= 8w(w + 1) - (w + 1) \\ &= (8w - 1)(w + 1) \end{aligned}$$

$$\begin{aligned} \text{i. } 8x^2 - 14x - 15 &= 8x^2 - 20x + 6x - 15 \\ &= 4x(2x - 5) + 3(2x - 5) \\ &= (4x + 3)(2x - 5) \end{aligned}$$

$$\begin{aligned} \text{j. } 27m^2 + 21m + 2 &= 27m^2 + 3m + 18m + 2 \\ &= 3m(9m + 1) + 2(9m + 1) \\ &= (3m + 2)(9m + 1) \end{aligned}$$

Problem Set 6

$$\begin{aligned} \text{a. } x^4 - 2x^2y - 99y^2 &= x^4 + 9x^2y - 11x^2y - 99y^2 \\ &= x^2(x^2 + 9y) - 11y(x^2 + 9y) \\ &= (x^2 - 11y)(x^2 + 9y) \end{aligned}$$

$$\begin{aligned} \text{b. } 14w^2 - 31w - 10 &= 14w^2 + 4w - 35w - 10 \\ &= 2w(7w + 2) - 5(7w + 2) \\ &= (2w - 5)(7w + 2) \end{aligned}$$

$$\begin{aligned} \text{c. } 32 + 4w^{20} - w^{40} &= 32 - 4w^{20} + 8w^{20} - w^{40} \\ &= 4(8 - w^{20}) + w^{20}(8 - w^{20}) \\ &= (4 + w^{20})(8 - w^{20}) \end{aligned}$$

$$\begin{aligned} \text{d. } 15x^2 - 16xy + 4y^2 &= 15x^2 - 6xy - 10xy + 4y^2 \\ &= 3x(5x - 2y) - 2y(5x - 2y) \\ &= (3x - 2y)(5x - 2y) \end{aligned}$$

$$\begin{aligned} \text{e. } x^2 + 15xy^2 + 36y^4 &= x^2 + 3xy^2 + 12xy^2 + 36y^4 \\ &= x(x + 3y^2) + 12y^2(x + 3y^2) \\ &= (x + 12y^2)(x + 3y^2) \end{aligned}$$

$$\begin{aligned} \text{f. } 4a^2 - 24ab + 35b^2 &= 4a^2 - 10ab - 14ab + 35b^2 \\ &= 2a(2a - 5b) - 7b(2a - 5b) \\ &= (2a - 7b)(2a - 5b) \end{aligned}$$

$$\begin{aligned} 9. \quad 1 - 59w^8 - 60w^{16} &= 1 + w^8 - 60w^8 - 60w^{16} \\ &= 1(1 + w^8) - 60w^8(1 + w^8) \\ &= (1 - 60w^8)(1 + w^8) \end{aligned}$$

$$\begin{aligned} h. \quad 2t^4 - 7t^2 - 30 &= 2t^4 + 5t^2 - 12t^2 - 30 \\ &= t^2(2t^2 + 5) - 6(2t^2 + 5) \\ &= (t^2 - 6)(t^2 + 5) \end{aligned}$$

$$\begin{aligned} i. \quad t^{10} + 16t^5 + 64 &= t^{10} + 8t^5 + 8t^5 + 64 \\ &= t^5(t^5 + 8) + 8(t^5 + 8) \\ &= (t^5 + 8)(t^5 + 8) \\ &= (t^5 + 8)^2 \end{aligned}$$

$$\begin{aligned} j. \quad 15m^2n^2 + 8mn + 1 &= 15m^2n^2 + 3mn + 5mn + 1 \\ &= 3mn(5mn + 1) + 1(5mn + 1) \\ &= (3mn + 1)(5mn + 1) \end{aligned}$$

Problem Set 7

$$a. \quad x^2 - 81 = (x - 9)(x + 9)$$

$$b. \quad 4w^2 - 9y^2 = (2w - 3y)(2w + 3y)$$

$$c. \quad 100 - 49x^{10} = (10 - 7x^5)(10 + 7x^5)$$

$$d. \quad y^{42} - 25x^2 = (y^{21} - 5x)(y^{21} + 5x)$$

$$e. \quad 49x^2 - 1 = (7x - 1)(7x + 1)$$

$$f. \quad 121 - 9w^4 = (11 - 3w^2)(11 + 3w^2)$$

$$g. \quad 36x^{18} - 25y^6 = (6x^9 - 5y^3)(6x^9 + 5y^3)$$

Problem Set 8

$$\begin{aligned} a. \quad t^3 - y^6 &= (t - y^2)[t^2 + ty^2 + (y^2)^2] \\ &= (t - y)(t^2 + ty^2 + y^4) \end{aligned}$$

$$\begin{aligned} b. \quad 27 - x^3 &= (3 - x)[3^2 + 3x + x^2] \\ &= (3 - x)(9 + 3x + x^2) \end{aligned}$$

$$\begin{aligned} c. \quad 8x^3 + 27y^3 &= (2x + 3y)[(2x)^2 - (2x)(3y) + (3y)^2] \\ &= (2x + 3y)(4x^2 - 6xy + 9y^2) \end{aligned}$$

$$\begin{aligned} \text{d. } 216 + 125m^6 &= (6 + 5m^2) \left[6^2 - 6(5m^2) + (5m^2)^2 \right] \\ &= (6 + 5m^2)(36 - 30m^2 + 25m^4) \end{aligned}$$

$$\begin{aligned} \text{e. } 8t^3 - 1 &= (2t - 1) \left[(2t)^2 + (2t)(1) + 1^2 \right] \\ &= (2t - 1)(4t^2 + 2t + 1) \end{aligned}$$

$$\begin{aligned} \text{f. } x^9 + 1000 &= (x^3 + 10) \left[(x^3)^2 + (x^3)(10) + 10^2 \right] \\ &= (x^3 + 10)(x^6 + 10x^3 + 100) \end{aligned}$$

$$\begin{aligned} \text{g. } 27x^6 + 125y^3 &= (3x^2 + 5y) \left[(3x^2)^2 - (3x^2)(5y) + (5y)^2 \right] \\ &= (3x^2 + 5y)(9x^4 - 15x^2y + 25y^2) \end{aligned}$$

Problem Set 9

- a. $x^2 + 81$ is prime b. $4w^2 + 9y^2$ is prime c. $100 + 49x^{10}$ is prime
d. $y^4 + 25x^2$ is prime e. $49x^2 + 49 = 49(x^2 + 1)$ f. $121 + 9w^4$ is prime

Problem Set 10

- a. $3x^2 - 33x + 84 = 3(x^2 - 11x + 28)$ b. $32w^4 - 18w^2 = 2w^2(16w^2 - 9)$
 $= 3(x - 4)(x - 7)$ $= 2w^2(4w - 3)(4w + 3)$
- c. $120x^4 + 120x^3 - 90x^2 = 30x^2(4x^2 + 4x - 3)$
 $= 30x^2[4x^2 - 2x + 6x - 3]$
 $= 30x^2[2x(2x - 1) + 3(2x - 1)]$
 $= 30x^2(2x + 3)(2x - 1)$
- d. $60t^7w - 320t^5w + 100t^3w = 20t^3w(3t^4 - 16t^2 + 5)$
 $= 20t^3w[3t^4 - 15t^2 - t^2 + 5]$
 $= 20t^3w[3t^2(t^2 - 5) - 1(t^2 - 5)]$
 $= 20t^3w[(3t^2 - 1)(t^2 - 5)]$
 $= 20t^3w(3t^2 - 1)(t^2 - 5)$

$$\begin{aligned}
 \text{e. } a^3 b^6 + 8b^3 &= b^3(a^3 + 8) \\
 &= b^3[(a+2)(a^2 - (a)(2) + 2^2)] \\
 &= b^3(a+2)(a^2 - 2a + 4)
 \end{aligned}$$

$$\text{f. } 15x^2 + 45x + 15 = 15(x^2 + 3x + 1)$$

$$\begin{aligned}
 \text{g. } x^6 y^4 z - 32x^4 y^5 z + 60x^2 y^6 z &= x^2 y^4 z(x^4 - 32x^2 y + 60y^2) \\
 &= x^2 y^4 z(x^4 - 2x^2 y - 30x^2 y + 60y^2) \\
 &= x^2 y^4 z[x^2(x^2 - 2y) - 30y(x^2 - 2)] \\
 &= x^2 y^4 z(x^2 - 30y)(x^2 - 2)
 \end{aligned}$$

$$\text{h. } x^4 y^2 + 9x^2 y^2 = x^2 y^2(x^2 + 9)$$