Group solutions involving polynomials and functions

1. a. $(x+6)(x-3)=x^2+3x-18$

   b. $(x-2)^2=(x-2)(x-2)$
   $=x^2-4x+4$

   c. $x(-x+5)=-x^2+5x$

2. a. $f(x)=(x+6)(x-3)$
   $f(2)=(2+6)(2-3)$
   $=(8)(-1)$
   $=-8$

   b. $g(x)=(x-2)^2$
   $g(2)=(2-2)^2$
   $=0^2$
   $=0$

   c. $h(x)=x(-x+5)$
   $h(2)=2(-2+5)$
   $=2(3)$
   $=6$

3. a. $f(x)=x^2+3x-18$
   $f(2)=2^2+3(2)-18$
   $=4+6-18$
   $=-8$

   b. $g(x)=x^2-4x+4$
   $g(2)=2^2-4(2)+4$
   $=4-8+4$
   $=0$

   c. $h(x)=-x^2+5x$
   $h(2)=-2^2+5(2)$
   $=-4+10$
   $=6$

4. a. The formulas used in part (a) were $(x+6)(x-3)$ and $x^2+3x-18$. In question 1 we showed that these formulas are equivalent, so it was inevitable that they would return the same value at 2 while working problems 2 and 3.

   The formulas used in part (b) were $(x-2)^2$ and x^2-4x+4. In question 1 we showed that these formulas are equivalent, so it was inevitable that they would return the same value at 2 while working problems 2 and 3.

   The formulas used in part (c) were $x(-x+5)$ and $-x^2+5x$. In question 1 we showed that these formulas are equivalent, so it was inevitable that they would return the same value at 2 while working problems 2 and 3.

   b. Since the formulas $x(-x+5)$ and $-x^2+5x$, they have to return the same value when x is replaced by 2. Likewise, they have to return the same value when x is replaced by, say, 5.

   When $x=5$, the formula $x(-x+5)$ clearly has a value of 0, so the formula $-x^2+5x$ must also have a value of 0. The only way that -5^2+5^2 has a value of 0 is if $-5^2=-25$; ergo, it has to be the case that -5^2 does indeed equal -25.

   Similarly, when working problem 3 (c) the value -2^2 ***<u>has</u>*** to be -4 or else the formulas $x(-x+5)$ and $-x^2+5x$ would not both evaluate to 6 when $x=2$.

5. a. $w(-4) = 3 - 8(-4)$
$= 3 + 32$
$= 35$

b. $w(x) = -4$
$3 - 8x = -4$
$-8x = -7$
$x = \frac{7}{8}$

The solution is $\frac{7}{8}$.

Check

$w\left(\frac{7}{8}\right) = 3 - \cancel{8}\left(\frac{7}{\cancel{8}}\right)$
$= 3 - 7$
$= -4$ ✓

c. $w(0) = 3 - 8(0)$
$= 3$

d. $w(x) = 0$
$3 - 8x = 0$
$-8x = -3$
$x = \frac{3}{8}$

The solution is $\frac{3}{8}$.

Check

$w\left(\frac{3}{8}\right) = 3 - \cancel{8}\left(\frac{3}{\cancel{8}}\right)$
$= 3 - 3$
$= 0$ ✓

6. a. $k(5) = 1$

b. $x = 8$ because $k(8) = 5$. The solution is 8.

7. a. $(w-6)(w^2+3w-1) = w^3 + 3w^2 - w - 6w^2 - 18w + 6$
$= w^3 - 3w^2 - 19w + 6$

b. $(x^2+1)(2x^3+4x^2-3x-2) = 2x^5 + 4x^4 - 3x^3 - 2x^2 + 2x^3 + 4x^2 - 3x - 2$
$= 2x^5 + 4x^4 - x^3 + 2x^2 - 3x - 2$

c. $(x^2-4x-2)(2x^2-4x+4) = 2x^4 - 4x^3 + 4x^2 - 8x^3 + 16x^2 - 16x - 4x^2 + 8x - 8$
$= 2x^4 - 12x^3 + 16x^2 - 8x - 8$

d. $(y-2)(y^2+2y+4) = y^3 + 2y^2 + 4y - 2y^2 - 4y - 8$
$= y^3 - 8$

e. $(x+6)^2 - (x+5) = (x+6)(x+6) - (x+5)$
$= x^2 + 12x + 36 - x - 5$
$= x^2 + 11x + 31$

f. $x(x-4) - (x-2)(x+3) = x^2 - 4x - (x^2 + x - 6)$
$= x^2 - 4x - x^2 - x + 6$
$= -5x + 6$