MTH 60 – Final Exam Review Key

1. a. The domain of f is $[-6,6)$.

   b. The range of f is $[-5,2]$.

   c. $f(-6)=-5$ and $f(6)$ is undefined

   d. $f(x)$ never equals 0!

2. a. None of the equations illustrate the commutative property of multiplication.

   b. $4+(5+9)=(4+5)+9$ illustrates the associative property of addition.

   c. $6\cdot(5+7)=6\cdot5+6\cdot7$ illustrates the distributive property.

   d. $4\cdot(3\cdot7)=(4\cdot3)\cdot(4\cdot7)$ is a contradiction; AKA a bunch of hooey.

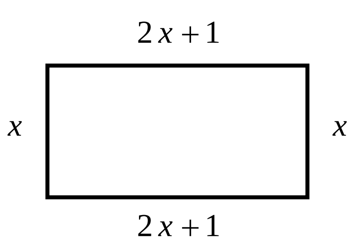
   e. $2+6\cdot8=2+48$ illustrates that multiplication comes before addition in order of operations.

3. a.
$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ 3\cdot V &= 3\cdot\frac{1}{3}\pi r^2 h \\ 3V &= \pi r^2 h \\ \frac{3V}{\pi r^2} &= \frac{\pi r^2 h}{\pi r^2} \\ \frac{3V}{\pi r^2} &= h \end{aligned}$$

   b.
$$\begin{aligned} P &= 2r + rA \\ P-2r &= rA \\ \frac{P-2r}{r} &= \frac{rA}{r} \\ \frac{P-2r}{r} &= A \end{aligned}$$

   c.
$$\begin{aligned} P &= 2r + rA \\ P &= r(2+A) \\ \frac{P}{2+A} &= \frac{r(2+A)}{(2+A)} \\ \frac{P}{2+A} &= r \end{aligned}$$

4. Let x represent the width (cm) of Hamid's rectangle. Then the length is $2x+1$. Since the perimeter of the rectangle is 46 cm, we have:



$$\begin{aligned} x+x+(2x+1)+(2x+1) &= 46 \\ 6x+2 &= 46 \\ 6x &= 44 \\ x &= \frac{44}{6} = \frac{22}{3} = 7\tfrac{1}{3} \\ 2x+1 &= \frac{47}{3} = 15\tfrac{2}{3} \end{aligned}$$

So Hamid's rectangle needs to be $7\tfrac{1}{3}\text{ cm}\times 15\tfrac{2}{3}\text{ cm}$.

Check

$7\tfrac{1}{3}+7\tfrac{1}{3}+15\tfrac{2}{3}+15\tfrac{2}{3}=44\tfrac{6}{3}=44+2=46$ ✓

5. a. $-4^2+(8-2)\cdot 3^2 = -4^2+6\cdot 3^2$
$= -16+6\cdot 9$
$= -16+54$
$= 38$

b. $3+4\cdot\frac{1}{-|-3+9|} = 3+4\cdot\frac{1}{-|6|}$
$= 3+4\cdot\frac{1}{-6}$
$= 3+-\frac{2}{3}$
$= \frac{7}{3}$

6. When $b = 7$ ft and $h = 17$ ft,

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(7 \text{ ft})(17 \text{ ft})$$
$$= 59.5 \text{ ft}^2$$

The area of the triangle is 59.5 ft^2.

7. a. $3x+1 \le 9$
$3x \le 8$
$\frac{3x}{3} \le \frac{8}{3}$
$x \le \frac{8}{3}$

The solution set to $3x+1\le 9$ is $\left(-\infty, \frac{8}{3}\right]$.

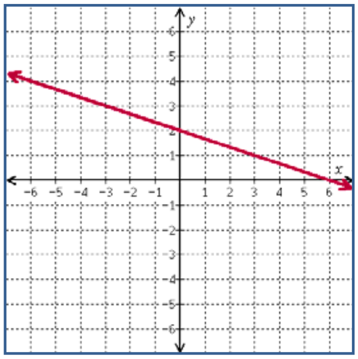
b. $5-(2-x) > 3$
$5-2+x > 3$
$3+x > 3$
$x > 0$

The solution set to $5-(2-x)>3$ is $(0,\infty)$.

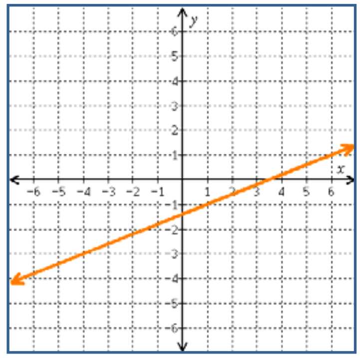
c. $5-3x \ge -22$
$-3x \ge -27$
$\frac{-3x}{-3} \le \frac{-27}{-3}$
$x \le 9$

The solution set to $5-3x \ge -22$ is $(-\infty, 9]$.

8. a.



b.



9. Let x be the regular price (\$) of a specialty bagel.

$$x - 0.25x = 1.35$$
$$0.75x = 1.35$$
$$x = \frac{1.35}{0.75}$$
$$x = 1.8$$

The regular price of a specialty bagel at Hole foods is \$1.80.

10. a.

$$3 + 2(5 - 2t) = 5t - (t + 13)$$
$$3 + 10 - 4t = 5t - t - 13$$
$$13 - 4t = 4t - 13$$
$$13 - 4t + 4t + 13 = 4t - 13 + 4t + 13$$
$$26 = 8t$$
$$\frac{26}{8} = \frac{8t}{8}$$
$$\frac{13}{4} = t$$

The solution set is $\left\{ \frac{13}{4} \right\}$.

b.

$$4 + 5\left[3x - 2(1 + x)\right] = 5x - 6$$
$$4 + 5[3x - 2 - 2x] = 5x - 6$$
$$4 + 5(x - 2) = 5x - 6$$
$$4 + 5x - 10 = 5x - 6$$
$$5x - 6 = 5x - 6$$
$$5x - 6 - 5x = 5x - 6 - 5x$$
$$-6 = -6$$

Identity!

The solution set is $\mathbb{R}$.

c.

$$\frac{5a + 2}{3} = \frac{3a - 1}{2}$$
$$2(5a + 2) = 3(3a - 1)$$
$$10a + 4 = 9a - 3$$
$$a = -7$$

The solution set is $\{-7\}$.

d.

$$-(-2x + 5) = -x - (4 - 3x)$$
$$2x - 5 = -x - 4 + 3x$$
$$2x - 5 = 2x - 4$$
$$2x - 5 - 2x = 2x - 4 - 2x$$
$$-5 = -4$$

Contradiction!

The solution set is $\{ \ \}$.

e. $$\frac{4}{x+2}=\frac{7}{x-4}$$

$$4(x-4)=7(x+2)$$

$$4x-16=7x+14$$ The solution set is $\{-10\}$.

$$-30=3x$$

$$-10=x$$

11. a. $3-2(t-7)-(8-t)=3-2t+14-8+t$
$=-t+9$

b. $4a(1-a)+(2a)^2=4a-4a^2+4a^2$
$=4a$

12. a. $m=\dfrac{-33-(-6)}{4-17}$
$=\dfrac{-27}{-13}$
$=\dfrac{27}{13}$
$x_1=17$
$y_1=-6$

$$y-y_1=m(x-x_1)$$
$$y-(-6)=\frac{27}{13}(x-17)$$
$$y+6=\frac{27}{13}x-\frac{459}{13}$$
$$y=\frac{27}{13}x-\frac{537}{13}$$

Check $(4,-33)$

$$-33=\frac{27}{13}\cdot\frac{4}{1}-\frac{537}{13}\ ?$$
$$-\frac{429}{13}=\frac{108}{13}-\frac{537}{13}\ ?$$
$$-\frac{429}{13}=-\frac{429}{13}\ \checkmark$$

The equation of the line is $y=\dfrac{27}{13}x-\dfrac{537}{13}$.

b. By inspection, the equation of the line is $x=\dfrac{1}{4}$.

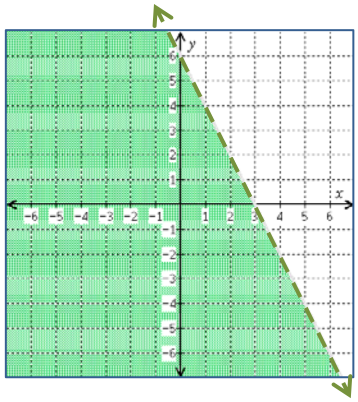
c. By inspection, the equation of the line is $y=0$.

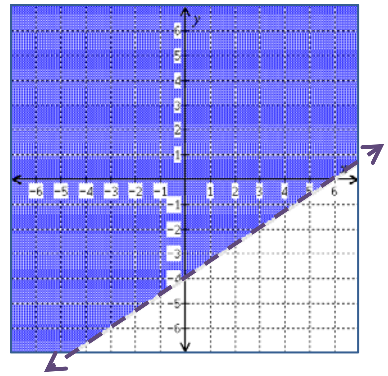
13. $4x-12y=18$
$4x=12y+18$
$4x-18=12y$
$\dfrac{4x-18}{12}=\dfrac{12y}{12}$
$\dfrac{4x}{12}-\dfrac{18}{12}=y$
$\dfrac{1}{3}x-\dfrac{3}{2}=y$

The slope of the line $4x-12y=18$ is $\dfrac{1}{3}$ so the slope of the perpendicular line is -3. Using $m=-3, x_1=8, y_1=0$ in point-slope we get:

$$y-y_1=m(x-x_1)$$
$$y-0=-3(x-8)$$
$$y=-3x+24$$

The equation of the perpendicular line is $y=-3x+24$.

14. a. 

b. 

15. The rise is 23.2 hours. The run is 8 pancakes

$$m = \frac{23.2 \text{ hours}}{8 \text{ pancakes}}$$
$$= 2.9 \, {}^{\text{hours}}\!/_{\text{pancake}}$$

That is, you feel bloated for an extra 2.9 hours for every one pancake you eat!

16. Please note that I'm only showing steps to help you find your mistake if you didn't get the correct answer.

a. $$-x^2 x^9 = -x^{2+9}$$
$$= -x^{11}$$

b. $$(3x)^3 = 3^3 x^3$$
$$= 27x^3$$

c. $$(4xy^5)^2 (3x) = 4^2 x^2 (y^5)^2 (3x)$$
$$= 16 \cdot 3x^{2+1} y^{5 \cdot 2}$$
$$= 48x^3 y^{10}$$

d. $$a^6 (a^6)^5 = a^6 a^{6 \cdot 5}$$
$$= a^{6+30}$$
$$= a^{36}$$

e. $$t^2 (t^5 t^8)^2 = t^t (t^{5+8})^2$$
$$= t^2 (t^{13})^2$$
$$= t^2 t^{13 \cdot 2}$$
$$= t^{2+26}$$
$$= t^{28}$$

f. $$w^7 + w^7 = (1+1)w^7$$
$$= 2w^7$$

g. $$(5x)^2 + 5x^2 = 5^2 x^2 + 5x^2$$
$$= 25x^2 + 5x^2$$
$$= (25+5)x^2$$
$$= 30x^2$$

i. $$(1+5)^2 = 6^2$$
$$= 36$$

17. a. $f(7) = -7^2$
$= -49$

b. $g(-8) = 3 - (-8)$
$= 3 + 8$
$= 11$

c. $k(8) = 19$

18. Let x be the number of licorice sticks Joe Moma could buy.

$$\begin{aligned} \frac{x}{12.36} &= \frac{3}{1.19} \\ 1.19x &= 3(12.36) \\ x &= \frac{37.08}{1.19} \\ &\approx 31.2 \end{aligned}$$

Assuming that Joe Moma could only buy licorice stick in bundles of 3, Joe could buy at most 30 licorice sticks.