Definition and most awesome fact

The equation $y = mx + b$ is called the ***slope-intercept form*** of a linear equation.

The equation of any non-vertical line can be written in this form. When the equation is written in this form, the number m **is the slope of the line** and the point $(0, b)$ **is the y-intercept of the line**.

Example 1

State the slope and y-intercept of the line with equation $y = -\frac{2}{5}x + 3$. Graph the line onto Figure 1.

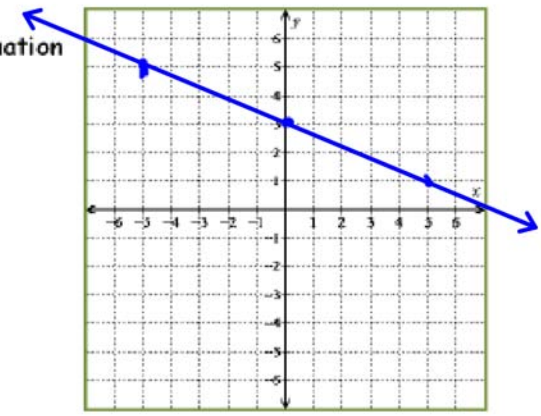


Figure 1: $y = -\frac{2}{5}x + 3$

$m = -\frac{2}{5}$

y-int: $(0, 3)$

$m: \frac{-2}{+5}$ $\frac{\text{down } 2}{\text{right } 5}$ $\frac{\text{up } 2}{\text{left } 5}$ $\frac{+2}{-5}$

Check $(5, 1)$

$1 = -\frac{2}{5}(5) + 3$?

$1 = -2 + 3$ ✓

Example 2

State the slope and y-intercept of the line with equation $y + 2x = -1$. Graph the line onto Figure 2.

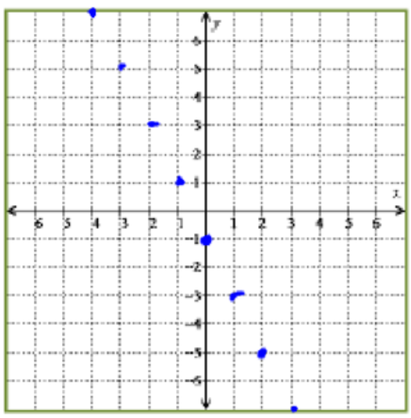


Figure 2: $y + 2x = -1$

$y + 2x = -1$

$y + 2x - 2x = -1 - 2x$

$y = -2x - 1$

$m = -2$

y-intercept: $(0, -1)$

Think thus: $\frac{-2}{+1}$ $\frac{\text{down } 2}{\text{right } 1}$ $\frac{+2}{-1}$ $\frac{\text{up } 2}{\text{left } 1}$

The slope is -2 and the y-intercept is $(0, -1)$.

Check $(2, -5)$

$-5 + 2(2) = -1$ ✓

Example 3

State the slope and y-intercept of the line with equation $y = -3$. Graph the line onto Figure 3.

$y = -3$

$y = 0x - 3$

$m = 0$

The slope is 0!

The y-intercept is $(0, -3)$.

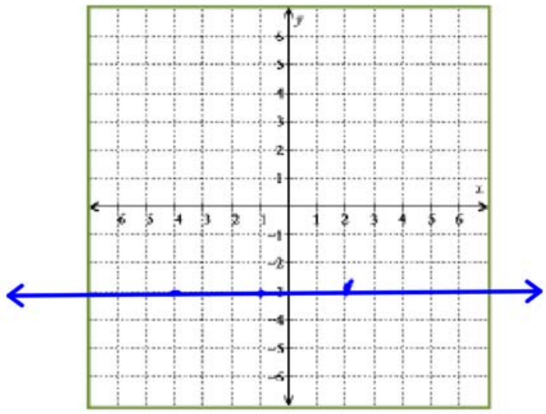


Figure 3: $y = -3$

Example 4

Find equations for the lines in figures 4 and 5.

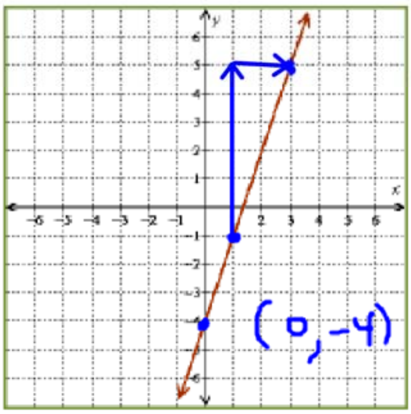


Figure 4

$m = \frac{6}{2}$
$= 3$

$(0, -4)$

The equation of the line is $y = 3x - 4$.

Check $(1, -1)$

$-1 = 3(1) - 4$? ✓

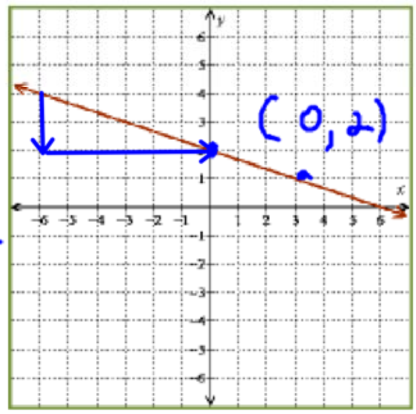


Figure 5

$m = \frac{-2}{6}$
$= -\frac{1}{3}$

$(0, 2)$

The equation is $y = -\frac{1}{3}x + 2$

Check $(3, 1)$

$1 = -\frac{1}{3}(3) + 2$?

$1 = -1 + 2$ ✓

$$y - y_1 = m(x - x_1)$$
$$\frac{y - y_1}{x - x_1} = \frac{m\cancel{(x - x_1)}}{\cancel{(x - x_1)}}$$
$$\frac{y - y_1}{x - x_1} = m$$

Point-slope

The equation of the line with slope m that passes through the point (x_1, y_1) can be found using the template $y - y_1 = m(x - x_1)$.

stays y # # stays x #

Example 1

Use point-slope to find the equation of the line which passes through the point $(2,7)$ with a slope of -6. Write the equation in slope-intercept form.

$x_1 = 2$
$y_1 = 7$
$m = -6$

$y - y_1 = m(x - x_1)$
$y - 7 = -6(x - 2)$
$y - 7 = -6x + 12$
$y - 7 + 7 = -6x + 12 + 7$
$y = -6x + 19$

Check $(2,7)$
$7 = -6(2) + 19$ ✓

The equation is $y = -6x + 19$

Example 2

Use point-slope to find the equation of the line which passes through the points $(-1,0)$ and $(9,-20)$ Write the equation in slope-intercept form.

$x_1 = -1$
$y_1 = 0$
$x_2 = 9$
$y_2 = -20$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-20 - 0}{9 - (-1)} = \frac{-20}{10} = -2$

The equation of the line is $y = -2x - 2$.

Point-slope

$y - y_1 = m(x - x_1)$
$y - 0 = -2(x - (-1))$
$y = -2(x + 1)$
$y = -2x - 2$

Switch-er-roo point-slope

$x_1 = 9$
$y_1 = -20$
$y - (-20) = -2(x - 9)$
$y + 20 = -2x + 18$
$y + 20 - 20 = -2x + 18 - 20$
$y = -2x - 2$

Example 3

Use point-slope to find the equation of the line in Figure 1. Write the equation in slope-intercept form.

$m = \frac{-2}{7}$

$x_1 = 2$

$y_1 = 3$

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{2}{7}(x - 2)$

$y - 3 = \frac{-2}{7}x + \frac{4}{7}$

$y - 3 + 3 = -\frac{2}{7}x + \frac{4}{7} + 3$

$y = -\frac{2}{7}x + \frac{25}{7}$

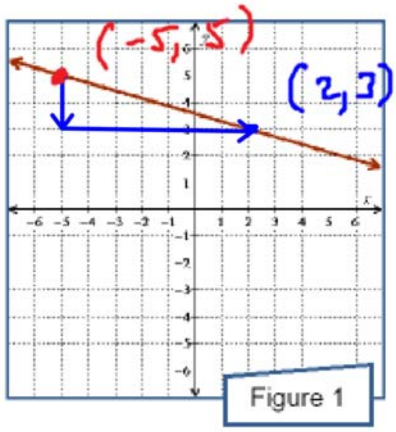


Figure 1

check (−5, 5)

$5 = -\frac{2}{7}(-5) + \frac{25}{7}$?

$5 = \frac{10}{7} + \frac{25}{7}$?

$5 = 35/7$ ✓

The equation is $y = -\frac{2}{7}x + \frac{25}{7}$.

Example 4

Why would it be kind of silly to use point-slope to find the equation of the line in Figure 2. What is the equation of the line?

We know that the y-intercept of the line is (0, −2), so once we determine the slope we can just write down the equation of the line using slope-intercept ($y = mx + b$).

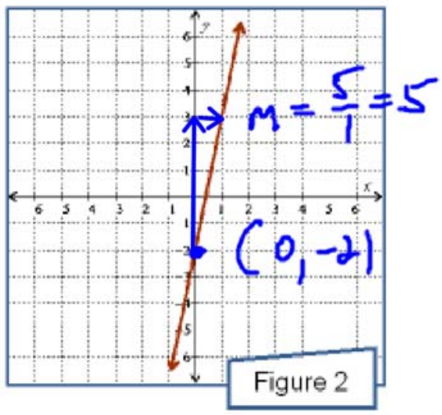


Figure 2

The equation is $y = 5x - 2$.

Example 5

with a slope of 3

Find the equation of the line through the point $(2,4)$ two different ways: once using point-slope and once using slope-intercept.

Using point-slope

$m=3$

$x_1=2$

$y_1=4$

$y-y_1=m(x-x_1)$

$y-4=3(x-2)$

$y-4=3x-6$

$y-4+4=3x-6+4$

$y=3x-2$

$m=3$, so the equation is $y=3x+b$ for some value of b.

Since $(2,4)$ satisfies the equation

$4=3(2)+b$

$4=6+b$

$4-6=6+b-6$

$-2=b$

The equation of the line is $y=3x-2$.

Example 6 $y=mx+b$

Use slope-intercept to find the equation of the line that passes through the point $(1,7)$ that is perpendicular to the line with equation $-3x-4y=6$.

Objective 1: Find the slope!

Relevant thought 1: perpendicular lines have opposite reciprocal slopes

relevant thought 2: I can figure out the slope of $-3x-4y=6$!

$-3x-4y=6$

$-3x-4y+3x=6+3x$

$-4y=6+3x$

$\frac{-4y}{-4}=\frac{3x+6}{-4}$

$y=\frac{3x}{-4}+\frac{6}{-4}$

$y=-\frac{3}{4}x-\frac{3}{2}$

The given line's slope is $-\frac{3}{4}$ so the $\perp$ line's slope is $4/3$.

$m=\frac{4}{3}$

$x_1=1$

$y_1=7$

We know that $y=\frac{4}{3}x+b$, so

$7=\frac{4}{3}(1)+b$

$7-\frac{4}{3}=b+\frac{4}{3}-\frac{4}{3}$

$17/3=b$

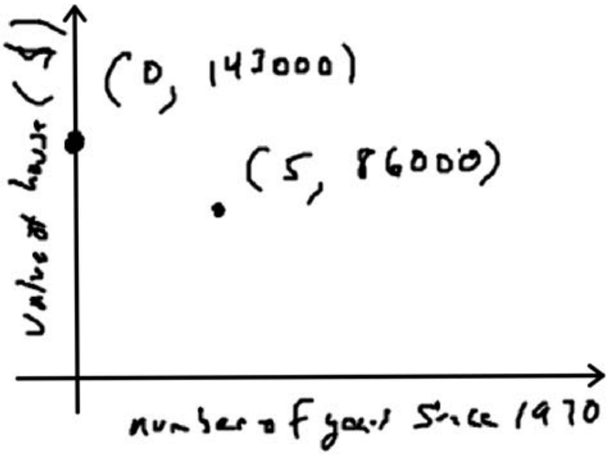
The equation of the perpendicular line is $y=\frac{4}{3}x+\frac{17}{3}$.

Example 8

The purchase of real-estate in Detroit was not a good investment anytime between 1968 and 2000. On January 1, 1970, the house at the corner of Livernois Ave and Tireman Street had a value of \$143,000. Five years later, the value had fallen to \$86,000! Let's define t to be the number of years that had passed since January 1, 1970. Let's also assume that the decline in the value of the house was linear.

$$y - y_1 = m(x - x_1)$$

Use point-slope to find the equation of the line and then use the equation to predict when the house's value had fallen to \$50,000.



$$m = \frac{-\$57000}{5\text{ year}} = -11400\ \$/\text{yr}$$

The value of the house decreased at the constant rate of 11,400 \$/yr.

$x_1 = 5$

$y_1 =$ `00

$m =$ 100

$$y - 86000 = -11400(x - 5)$$
$$y - 86000 = -11400x + 57000$$
$$y - 86000 + 86000 = -11400x + 57000 + 86000$$
$$y = -11400x + 143000$$

. . was value \$50,000?

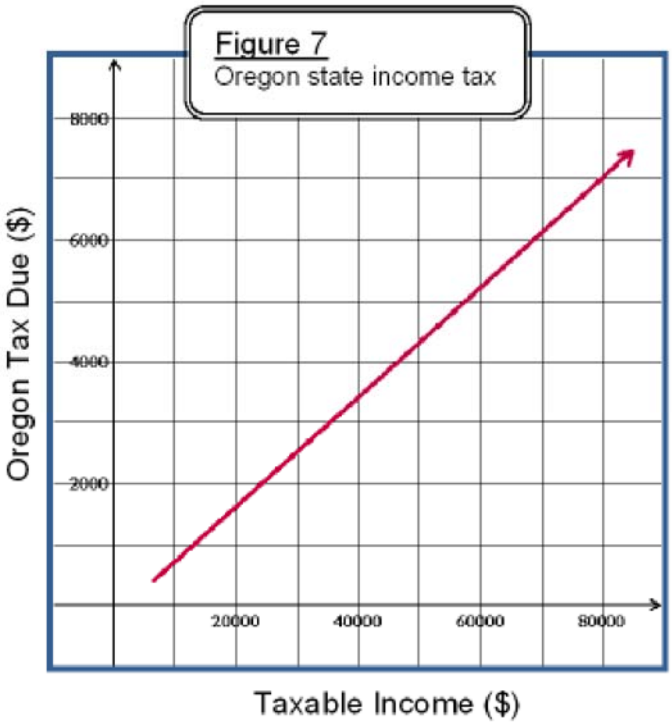
$$50000 = -11400x + 143000$$
$$50000 - 143000 = -11400x + 143000 - 143000$$
$$-93000 = -11400x$$
$$\frac{-93000}{-11400} = \frac{-11400x}{-11400}$$
$$8.158 \approx x$$

The value was \$50,000 early in 1978.

Example 9

If your taxable income in Oregon is greater than \$6,500, then the amount of income tax you have to pay for a given year is \$403 plus 9% of the amount of taxable income you have over \$6,500. The formula for this tax is $T = 403 + .09(I - 6500)$ where T is the tax owed (\$) by a person whose taxable income is I dollars. A graph of the equation is shown in Figure 7.

Use the graph to estimate the taxable income of a person who had to pay \$5,500 in Oregon state income tax for 2005. Then use the formula to find the exact taxable income of a person who paid \$5,500 in Oregon state income tax for 2006.

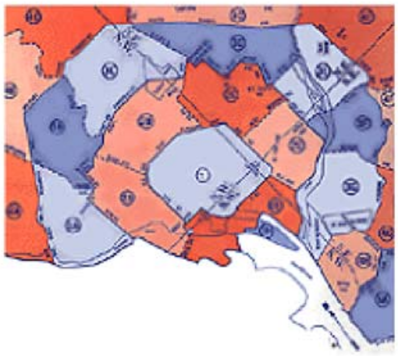


Group work problems

1. Use point-slope to find the equation of the line which passes through the points $(5,3)$ and $(-6,14)$ Write the equation in slope-intercept form.

2. Use point-slope to find the equation of the line which passes through the point $(-3,5)$ that is also perpendicular to the line with equation $6x+2y=11$. Write the equation in slope-intercept form.

3. The cost of a taxi cab rides in Washington, D.C. is dependent upon the number of zones you pass through. If x is the number of zones past your original zone you enter and y is the cost of the cab ride in \$, then the equation for the cab ride is $y=2.3x+6.5$.



   a. What is the slope of this line (including unit). Interpret the slope as a rate of change.

   b. What is the y-intercept of this line (including unit). Interpret the y-intercept in the context of this question; i.e., what does it tell you about the cost of cab rides in Washington, D.C.

   FYI ... This problem is mostly factual – i.e., this really is for the most part how cab fares are determined in D.C.

4. Use point-slope to find the equation of the line that passes through the point $(8.72,-2.84)$ that is parallel to the line that passes through the points $(-14.86,7.06)$ and $(78.10,7.06)$.

5. Use slope-intercept to find the equation of the line that passes through the points $(9,2)$ and $(-5,30)$.

6. A certain line passes through the point $(9,10)$. This line also happens to be parallel to the line that passes through the points $(-2,7)$ and $(1,-8)$. Use slope-intercept to find the equation of this line.