Point-slope group work problems

1. Use point-slope to find the equation of the line which passes through the points $(5,3)$ and $(-6,14)$ Write the equation in slope-intercept form.

The first thing we need to determine is the slope of the line.

	x	y	
x_2	5	3	y_2
x_1	-6	14	y_1

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-14}{5-(-6)} = \frac{-11}{11}$$

Now we can use the point-slope form of the equation of the line with $m = -1$, $x_1 = -6$, and $y_1 = 14$.

$$y - y_1 = m(x - x_1)$$
$$y - 14 = -1(x - (-6))$$
$$y - 14 = -1(x + 6)$$
$$y - 14 = -x - 6$$
$$y = -x + 8$$

Check

Does $(5,3)$ satisfy $y = -x + 8$?

$5 = -3 + 8$?

Ummm ... Yep!

The equation of the line is $y = -x + 8$.

2. Use point-slope to find the equation of the line which passes through the point $(-3,5)$ that is also perpendicular to the line with equation $6x + 2y = 11$. Write the equation in slope-intercept form.

The first thing we need to determine is the slope of the perpendicular line. Let's begin by finding the slope of the line $6x + 2y = 11$ after writing the equation in slope-intercept form.

$$6x + 2y = 11$$
$$2y = -6x + 11$$
$$\frac{2y}{2} = \frac{-6x + 11}{2}$$
$$y = \frac{-6x}{2} + \frac{11}{2}$$
$$y = -3x + \frac{11}{2}$$

The slope of the given line is -3, so the slope of the perpendicular line is $\frac{1}{3}$. (Perpendicular lines have opposite reciprocal slopes.)

We can use the point-slope form of the equation of the line with $m = \frac{1}{3}$, $x_1 = -3$, and $y_1 = 5$.

$$y - y_1 = m(x - x_1)$$
$$y - 5 = \frac{1}{3}(x - (-3))$$
$$y - 5 = \frac{1}{3}(x + 3)$$
$$y - 5 = \frac{1}{3}x + 1$$
$$y = \frac{1}{3}x + 6$$

The equation of the perpendicular line is $y = \frac{1}{3}x + 6$.

NOTE: The best was to check this answer is with a graph. I will have a graph available for you on a problem like this when you take your test.

3. The cost of a taxi cab rides in Washington, D.C. is dependent upon the number of zones you pass through. If x is the number of zones past your original zone you enter and y is the cost of the cab ride in \$, then the equation for the cab ride is $y = 2.3x + 6.5$.

   a. What is the slope of this line (including unit). Interpret the slope as a rate of change.

   The slope of the line is $2.3\ ^{\$}/_{\text{zone}}$. The rate of $2.3\ ^{\$}/_{\text{zone}}$ means that the total cost of the cab ride increases by \$2.30 for each additional zone through which you ride.

   b. What is the y-intercept of this line (including unit). Interpret the y-intercept in the context of this question; i.e., what does it tell you about the cost of cab rides in Washington, D.C.

   The y-intercept of this line is \$6.50. This means that if your cab ride is confined to the zone in which you board, your cab ride costs \$6.50.

4. Find the equation of the line that passes through the point $(8.72, -2.84)$ that is parallel to the line that passes through the points $(-14.86, 7.06)$ and $(78.10, 7.06)$.

   The slope of the given line is:

$$m = \frac{7.06 - 7.06}{78.10 - (-14.86)}$$
$$= \frac{0}{92.96}$$
$$= 0$$

   Horizonatal lines have a slope of 0, so the parallel line is also horizontal. Horizontal lines have equations of for $y = k$. So the equation of the parallel line is $y = -2.84$, Note that you could have come up with this equation using the point-slope form of the equation of a line.

5. Solution only.

The slope of the line is :

$$m = \frac{2-30}{9-(-5)}$$
$$= \frac{-28}{14}$$
$$= -2$$

From the slope we know that the equation is $y = -2x + b$. Using the point $(9,2)$ as (x,y) we get:

$$2 = -2(9) + b$$
$$20 = b$$

So the equation of the line is $y = -2x + 20$.

Checking the point $(-5,30)$ ...

$30 = -2(-5) + 20$?

$30 = 10 + 20$ ✔

6. Solution only.

The slope of the line through the points $(-2,7)$ and $(1,-8)$ is:

$$m = \frac{-8-7}{1-(-2)}$$
$$= \frac{-15}{3}$$
$$= -5$$

Since parallel lines have equal slope we know that the equation we are trying to find is $y = -5x + b$. Using the point $(9,10)$ as (x,y) we get:

$$10 = -5(9) + b$$
$$55 = b$$

So the equation of the line is $y = -5x + 55$.

Checking the point $(9,10)$ ...

$10 = -5(9) + 55$?

$10 = -45 + 55$ ✔