**Example 1**

For each of figures 1-4 state the scale on each axis and use the concept of rise over run to find the slope of each line.

$m = -2/3$

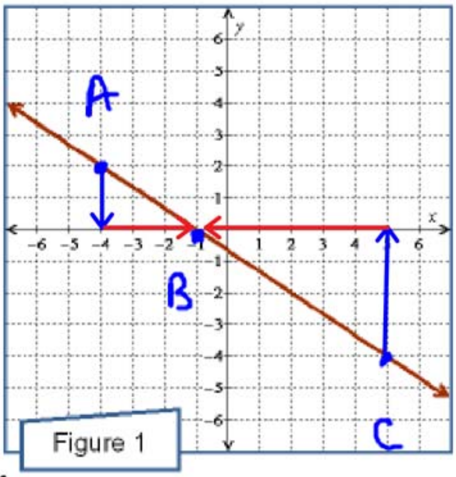


Figure 1

| $x$-scale | $y$-scale | slope |
|---|---|---|
| 1 | 1 | $-2/3$ |

From A to B

rise = −2    run = +3

$\frac{\text{rise}}{\text{run}} = \frac{-2}{+3} = -2/3$

From C to B

rise = +4    run = −6

$\frac{\text{rise}}{\text{run}} = \frac{+4}{-6} = -2/3$

The slope of this line is $-\frac{2}{3}$

you can interpret this two ways

$\frac{-2}{3} \rightarrow \frac{\text{Down } 2}{\text{Right } 3}$ | $\frac{2}{-3} \rightarrow \frac{\text{up } 2}{\text{left } 3}$

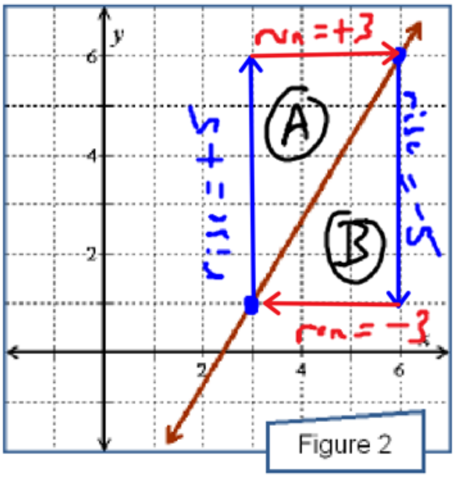


Figure 2

| $x$-scale | $y$-scale | slope |
|---|---|---|
| 1 | 1 | $5/3$ |

option A

$M = \frac{\text{rise}}{\text{run}} = \frac{+5}{+3} = 5/3$

option B

$M = \frac{\text{rise}}{\text{run}} = \frac{-5}{-3} = 5/3$

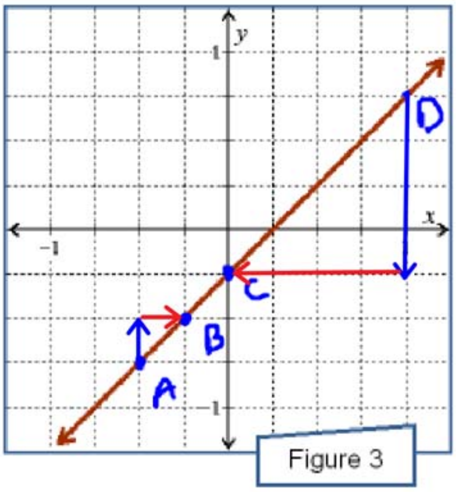


Figure 3

| $x$-scale | $y$-scale | slope |
|---|---|---|
| 1/4 | 1/4 | 1 |

From A to B

$$m = \frac{\text{rise}}{\text{run}} = \frac{+1/4}{+1/4} = 1$$

From D to C

$$m = \frac{\text{rise}}{\text{run}} = \frac{-1}{-1} = 1$$

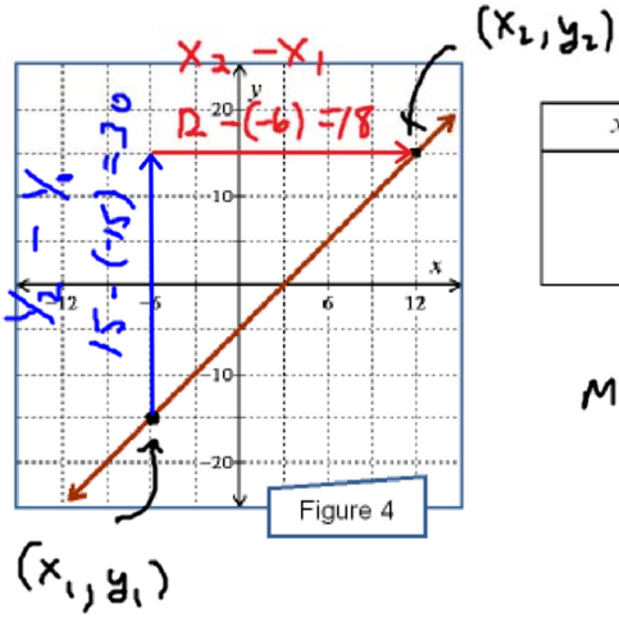


Figure 4

| $x$-scale | $y$-scale | slope |
|---|---|---|
| 3 | 5 | 5/3 |

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{15-(-15)}{12-(-6)} = \frac{30}{18} = 5/3$$

**Definition**

The ***slope*** of the line connecting the point $(x_1, y_1)$ and $(x_2, y_2)$ where $y_1 \neq y_2$ is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Slope is commonly referred to as "rise over run" where a positive rise means you go up, a negative rise means you go down, a positive run means you go right, and a negative run means you go left.

**Example 2**

Find the slope of the line connecting the points $(6,-3)$ and $(-4,2)$.

**Option A**

P $x_1 = 6$ $y_1 = -3$

Q $x_2 = -4$ $y_2 = 2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-3)}{-4-6} = \frac{5}{-10} = -\frac{1}{2}$$

**Option B**

Q $x_1 = -4$ $y_1 = 2$

P $x_2 = 6$ $y_2 = -3$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$M = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{-3-2}{6-(-4)} = \frac{-5}{10} = -\frac{1}{2}$$

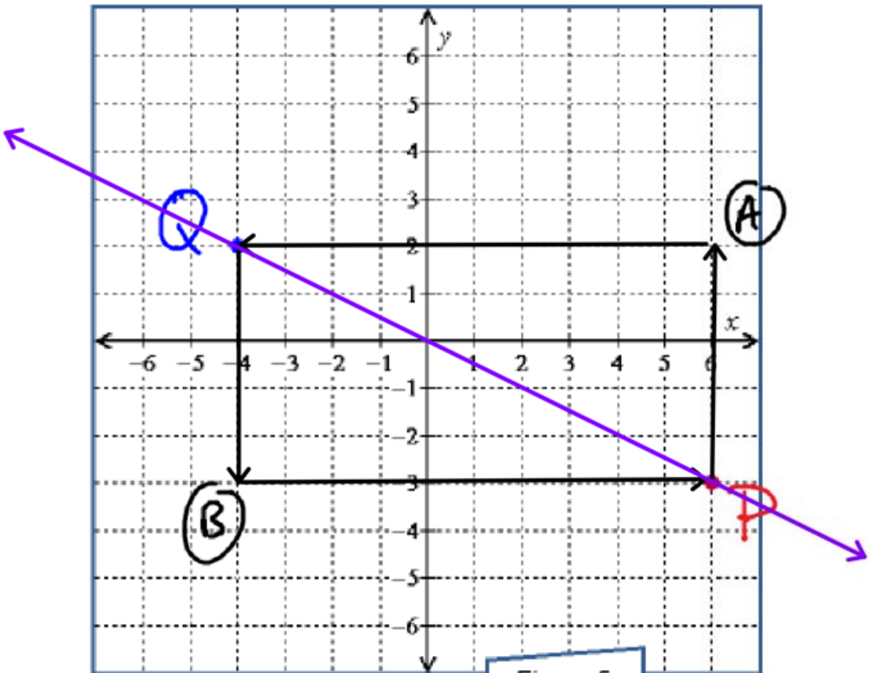



Figure 5

**Example 3**

Find the slope of the line connecting the points $(12,-5)$ and $(-6,10)$.

**Option A**

$x_1 = 12 \qquad y_1 = -5$

$x_2 = -6 \qquad y_2 = 10$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{10-(-5)}{-6-12}$$

$$= \frac{15}{-18}$$

$$= -5/6$$

**Option B**

$x_1 = -6 \qquad y_1 = 10$

$x_2 = 12 \qquad y_2 = -5$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-5-10}{12-(-6)}$$

$$= \frac{-15}{18}$$

$$= -5/6$$

Any freaking way you look at it, the slope is $-5/6$!

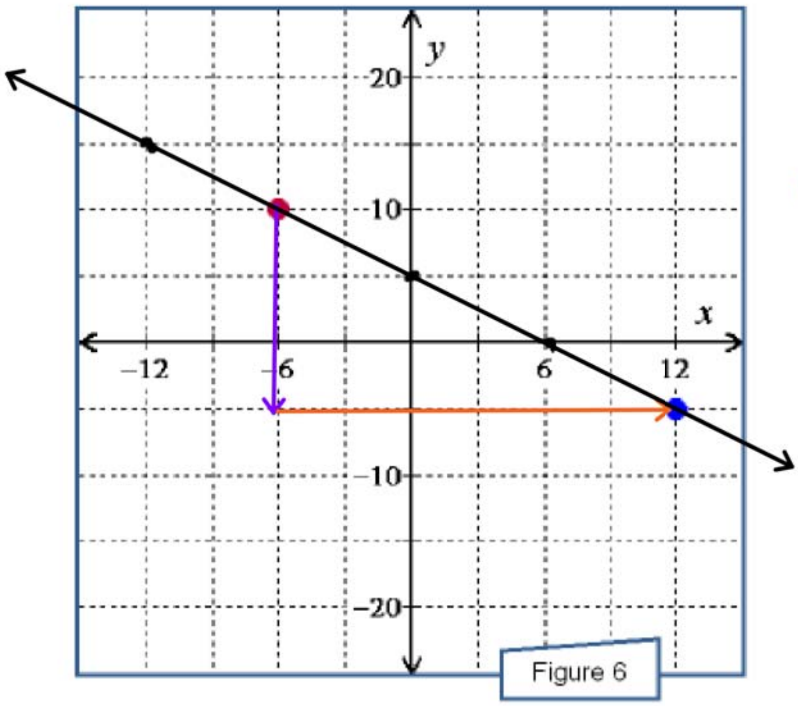



Figure 6

rise = −15

run = +18

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{-15}{+18}$$

$$= -5/6$$

**Example 5**

Find 2 points on the line $y = \frac{2}{7}x + 4$ and use those points to determine the slope of the line.

| | | | |
|---|---|---|---|
| $y_2$ | 4 | 6 | $y_1$ |
| $x_2$ | 0 | 7 | $x_1$ |

$\underline{x = 0}$

$y = \frac{2}{7}(0) + 4$

$= 4$

$(0, 4)$

$(x_2, y_2)$

$\underline{x = 7}$

$y = \frac{2}{7}(7) + 4$

$= 2 + 4$

$= 6$

$(7, 6)$

$(x_1, y_1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 6}{0 - 7} = \frac{-2}{-7} = 2/7$$

**Example 6**

Find 2 points on the line $y = -9x + 7$ and use those points to determine the slope of the line.

| | | | |
|---|---|---|---|
| $y_2$ | -2 | 7 | $y_1$ |
| $x_2$ | 1 | 0 | $x_1$ |

$\underline{x = 0}$

$y = 7$

$(0, 7)$

$(x_1, y_1)$

$\underline{x = 1}$

$y = -2$

$(1, -2)$

$(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 7}{1 - 0} = \frac{-9}{1} = -9$$

**Example 7**

Find 2 points on the line $x = -2y + 7$ and use those points to determine the slope of the line.

| | | | |
|---|---|---|---|
| $y_2$ | 1 | 0 | $y_1$ |
| $x_2$ | 5 | 7 | $x_1$ |

$\underline{y = 0}$

$x = 0 + 7$

$= 7$

$(7, 0)$

$(x_1, y_1)$

$\underline{y = 1}$

$x = -2 + 7$

$= 5$

$(5, 1)$

$(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{5 - 7} = \frac{1}{-2} = -\frac{1}{2}$$

**Example 8**

Find the slope of the lines in figures 7 and 8.

Horizontal lines always have a slope of 0.

Vertical lines have undefined slope

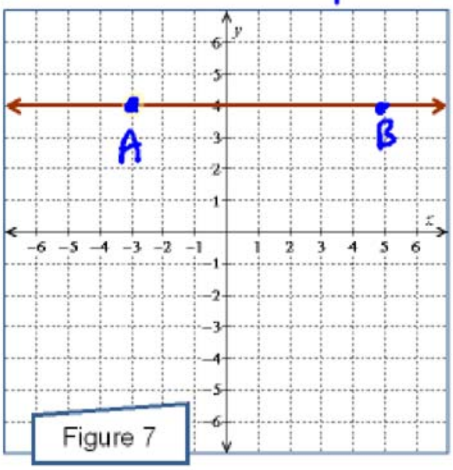



Figure 7

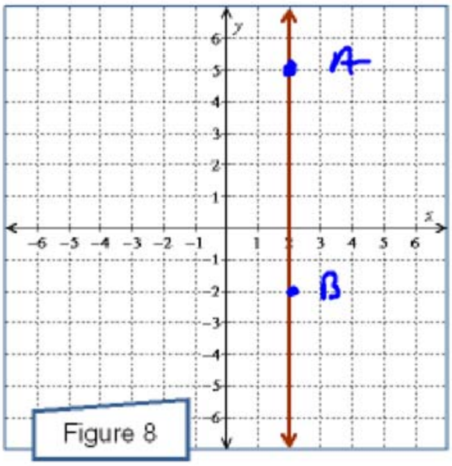



Figure 8

$A(-3,4) = (x_1,y_1)$

$B(5,4) = (x_2,y_2)$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{4-4}{5-(-3)} = \frac{0}{8} = 0$$

$A(2,5)$ $(x_1,y_1)$ $B(2,-2)$ $(x_2,y_2)$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{-2-5}{2-2} = -7/0$$ ← This is undefined!

**Example 9**

Draw two lines onto Figure 9. One line should pass through the point $(0,-4)$ and the other should pass through the point $(0,2)$. Both lines should have a slope of $-2$. What else do the two lines have in common?

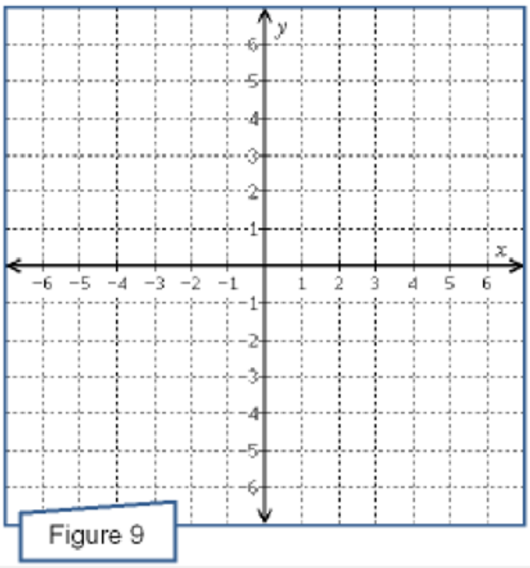


Figure 9

**Example 10**

Find the slope of each of the lines in figures 10 and 11. In each case, state which of the two lines has the greater slope.

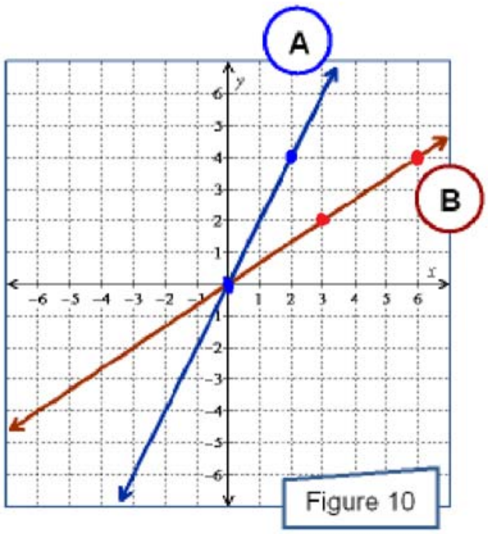



Figure 10

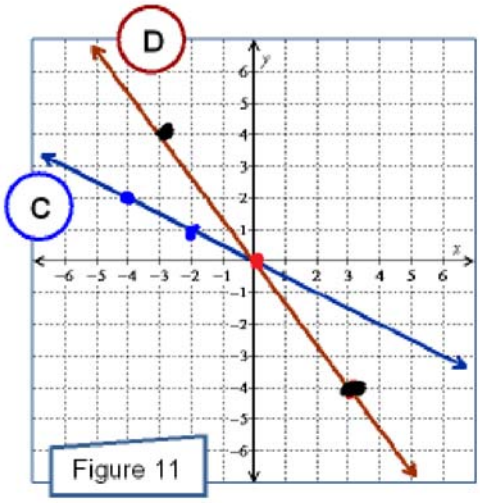



Figure 11

line A

$m = \frac{+4}{+2}$

$= 2$

line B

$m = \frac{+2}{+3}$

$= 2/3$

line C

$m = \frac{-1}{+2}$

$= -\frac{1}{2}$

line D

$m = \frac{-4}{+3}$

$= -4/3$

$-4/3 < -1/2$

True or False: The steeper the line the greater the slope.

True or False: The steeper the line the greater the absolute value of the slope.

**Example 11**

One of the lines in Figure 12 has a slope of $5$, one has a slope of $5/4$, one has a slope of $4/5$, one has a slope of $-1$, and one has a slope of $-2/3$. decide which line is which. Please note that scales and a grid have deliberately been omitted - the purpose of this problem is for you to *compare* the slopes of the lines, *not calculate* the slopes of the lines

negative slope    positive slope

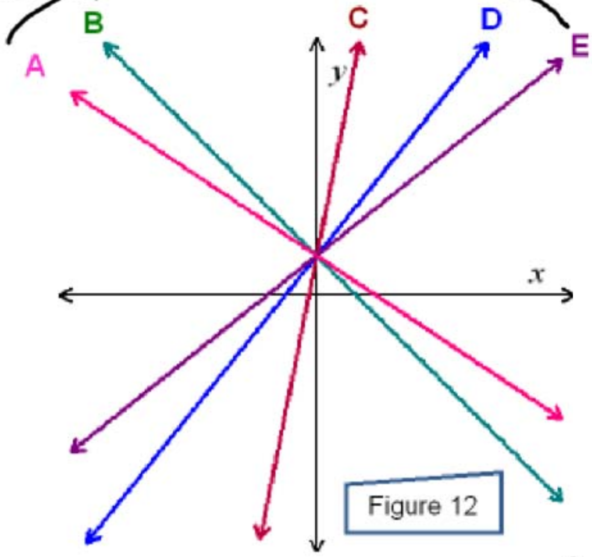


Figure 12

Table R

| Slope | Line |
|---|---|
| $5$ | C |
| $5/4$ | D |
| $4/5$ | E |
| $-1$ | B |
| $-2/3$ | A |

B is steeper.
$|-1| > |-2/3|$

**Example 12**

Graph onto Figure 13 the lines with equations $y=\frac{3}{2}x-4$ and $y=-\frac{2}{3}x+1$. What do you observe about the lines? What is the slope of each line? How are the slopes arithmetically related?

These lines are perpendicular!
Their slopes are opposite reciprocals

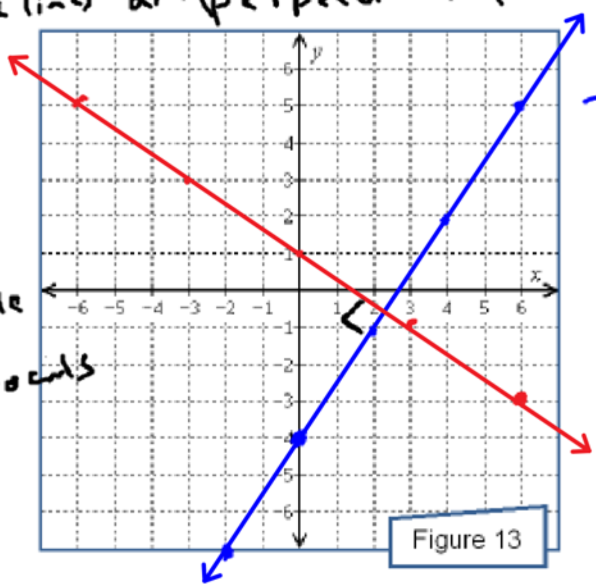


Figure 13

$y=\frac{3}{2}x-4$

y-intercept: $(0,-4)$

$m = 3/2$   $\frac{\text{up } 3}{\text{right } 2}$   $\frac{\text{down } 3}{\text{left } 2}$

Check $(4,2)$

$2 = \frac{3}{2}(4) - 4$?

$2 = 6 - 4$ ✓

$y = -\frac{2}{3}x + 1$

y-intercept: $(0,1)$

$m = -2/3$   $\frac{\text{Down } 2}{\text{Right } 3}$   $\frac{\text{up } 2}{\text{left } 3}$

Check $(6,-3)$

$-3 = -\frac{2}{3}(6) + 1$?

$-3 = -4 + 1$ ✓

$\ell_7 \perp \ell_8$

perpendicular



**Facts Jacks**

Two non-vertical lines are parallel if and only if they have equal slope.

If neither line is vertical, two lines are perpendicular if and only if the have opposite reciprocal slopes.

**Example 13**

Determine if each given pair of lines is parallel, perpendicular, or neither.

Pair 1: $\ell_1$ is the line through the points $(9,7)$ and $(-2,29)$.

$\ell_2$ is the line through the points $(-1,10)$ and $(-3,14)$.

$\ell_1 \parallel \ell_2$

$\ell_1$

$x_1 = 9$, $y_1 = 7$, $x_2 = -2$, $y_2 = 29$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{29-7}{-2-9} = \frac{22}{-11} = -2$$

$\ell_2$

$x_1 = -1$, $y_1 = 10$, $x_2 = -3$, $y_2 = 14$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14-10}{-3-(-1)} = \frac{4}{-2} = -2$$

These lines are Parallel!

Pair 2: $\ell_3$ is the line through the points $(-9,1)$ and $(6,-4)$.

$\ell_4$ is the line through the points $(2,5)$ and $(3,2)$.

$\ell_3$

$x_1 = -9$, $y_1 = 1$, $x_2 = 6$, $y_2 = -4$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-1}{6-(-9)} = \frac{-5}{15} = -1/3$$

$\ell_4$

$x_1 = 2$, $y_1 = 5$, $x_2 = 3$, $y_2 = 2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{3-2} = -3$$

These lines are neither Parallel nor Perpendicular!

**Example 15**

Use algebra to determine the value of $y$ that will make the line through the points $(1, y)$ and $(-3, -5)$ perpendicular to the line in Figure 14. Graph the line to verify your answer.

The slope of the given line is $-2/3$ so the slope of the perpendicular line is $3/2$.

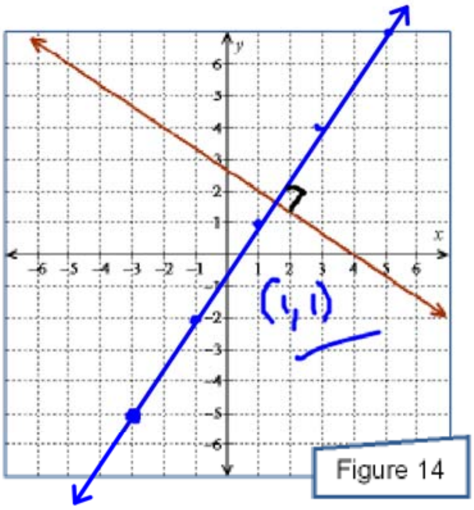


Figure 14

Perpendicular line

$x_1 = -3$

$y_1 = -5$

$x_2 = 1$

$y_2 = y$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3}{2}$$

$$\frac{y-(-5)}{1-(-3)} = \frac{3}{2}$$

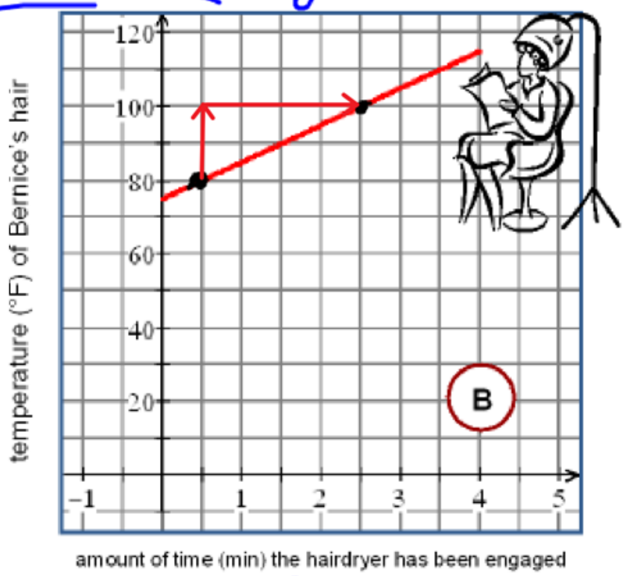
$$\frac{y+5}{4} = \frac{3}{2}$$

$$4\left(\frac{y+5}{4}\right) = 4\left(\frac{3}{2}\right)$$

$$y = 1 \leftarrow y + 5 = 6$$

**Example 15**

Bernice needed to touch up her greys. Bernice's stylist Bernard convinced her to try life as a blond for a month. The bleach Bernard used required Bernice to sit under the hairdryer for several minutes. The graph in Figure B shows the temperature (degrees Fahrenheit) of Bernice's hair for the first 4 minutes after the dryer was turned on. Find the slope of the line segment - including unit - and interpret the slope as a rate of change.

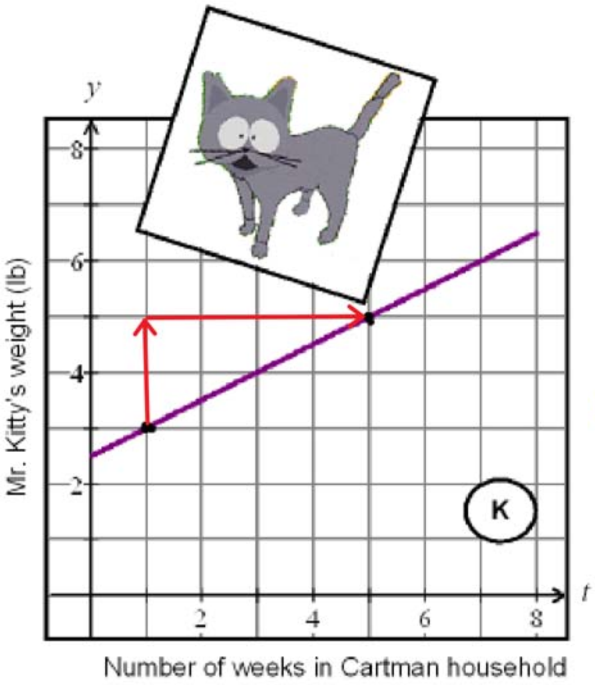



$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{20°F}{2 \text{ min}}$$

$$= 10 \text{ °F/min}$$

For the first four minutes under the dryer, the temperature of the top of Bernice's head rose at the constant rate of 10 °F/min.




**Example 16**

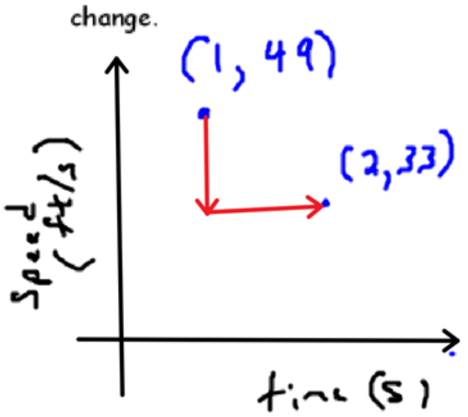
Figure K shows a graph of Mr. Kitty's weight (lb) $t$ weeks after the Cartmans brought him home. Calculate the slope of the line segment – *including unit* – and interpret the slope as a rate of change.

$$m = \frac{\text{rise}}{\text{run}} = \frac{+2 \text{ lbs}}{+4 \text{ weeks}} = \frac{1}{2} \text{ lb/wk}$$

For the first eight weeks in the Cartman household, Mr. Kitty gained weight at the constant rate of $\frac{1}{2}$ lb/wk.

**Example 17**

Suppose that you have a line where the $y$-coordinates are the speed (ft/s) at which a rolly-polly is travelling and the $x$-coordinates are the number of seconds that have passed since T-Bone flung the rolly-polly into the air. After 1 second the bug is travelling at 49 ft/s and after 2 seconds it's travelling at 33 ft/s. Find the slope of the line (including unit!) and interpret the slope as a rate of change.




$$m = \frac{\text{rise}}{\text{run}} = \frac{-16 \text{ ft/s}}{1 \text{ s}} = -16 \frac{\text{ft/s}}{\text{s}}$$

At least for that one second, the speed of the rolly-polly decreased at the constant rate of 16 $\frac{\text{ft/s}}{\text{s}}$

$\text{ft/s}^2$

$$\frac{2/3}{3} = \frac{2/3}{3/1} = \frac{2}{3} \times \frac{1}{3}$$

**Example 18**

The rate at which the Murphy's bed leaks depends upon the total weight of the people lying upon it. This relationship is linear over the interval shown in Table 1. Find the slope of this line (including unit!).

Table 1: Analysis of the leak in the Murphy's water bed

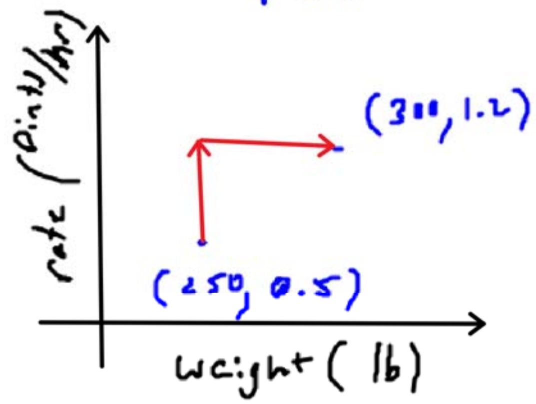
| Total weight on water bed (lb) | Rate at which water leaks (pints/hr) |
|---|---|
| 250 | 0.5 |
| 300 | 1.2 |
| 350 | 1.9 |
| 400 | 2.6 |

run: +50 lb

rise: +.7 pints/hr

"x" run

"y" rise




$$m = \text{rise}/\text{run}$$

$$= \frac{+0.7 \text{ pints/hr}}{+50 \text{ lbs}}$$

$$= \frac{7}{500} \; \frac{\text{pints/hr}}{\text{lb}}$$

$$3\overline{)21}\ \ 7$$

$$3\overline{)1500}\ \ 500$$

$$m = \frac{+2.1 \text{ pints/hr}}{+150 \text{ lb}}$$

$$= \frac{21}{1500} \; \frac{\text{pints/hr}}{\text{lb}}$$

$$= \frac{7}{500} \; \frac{\text{pints/hr}}{\text{lb}}.$$