Definitions

A solution to an equation with two variables, x and y, is an ***ordered pair*** (a,b) where a and b are real numbers with the property that if $x=a$ and $y=b$ the equation is true. The number a is called the ***x-coordinate*** of the ordered pair and the number b is called the ***y-coordinate*** of the ordered pair.

Example 1

What are the x and y coordinates of the ordered pair $(9,-2)$. Is the ordered pair a solution to the equation $3x+4y=10+x$?

Example 2

What is the ordered pair with an x-coordinate of -3 that satisfies the equation $y=\frac{1}{2}x-1$

Example 3

What is the ordered pair with a y-coordinate of -3 that satisfies the equation $y=\frac{1}{2}x-1$

Example 3

Complete Table 1.

Table 1: $y = -3x - 4$

x	y
2	
	2

Example 4

Write the missing values into Table 2 so that each implied ordered pair is a solution to the equation $x + y = 4$. Plot the four ordered pairs onto Figure 1.

Table 2: $x + y = 4$

x	y
0	
	0
6	
	2
−1.5	

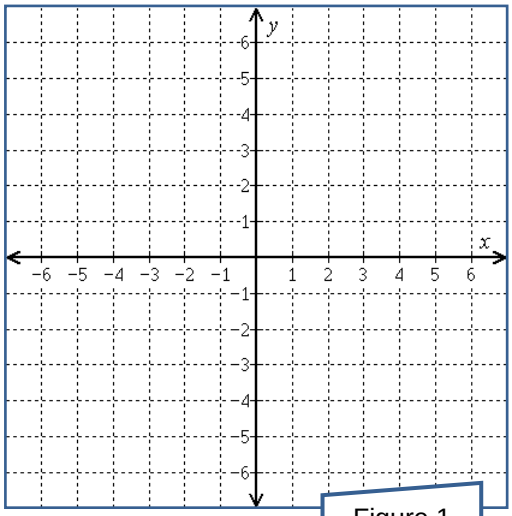

Figure 1

Example 5

The sum of the the coordinates of each point on the line in Figure 2 is always the same number. What is this constant sum?

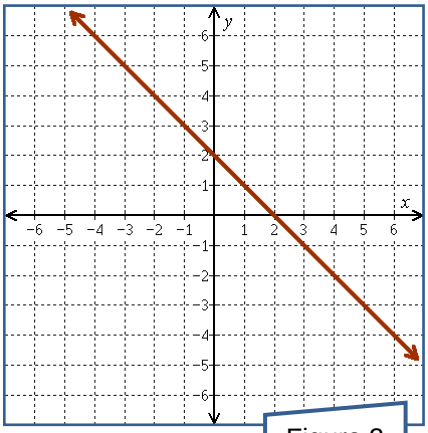

Figure 2

Table 3

point	ordered pair	$x + y$

Example 6

Several points are shown in Figure 3. State the ordered pair associated with each point and where in the coordinate plane the point lies; assume that both coordinates of each point are intergers. Which point, **B** or **F**, is a solution to the equation $3x - 2y = 8$?

Table 4

Point	Coordinates	Location
A		
B		
C		
D		
E		
F		
G		

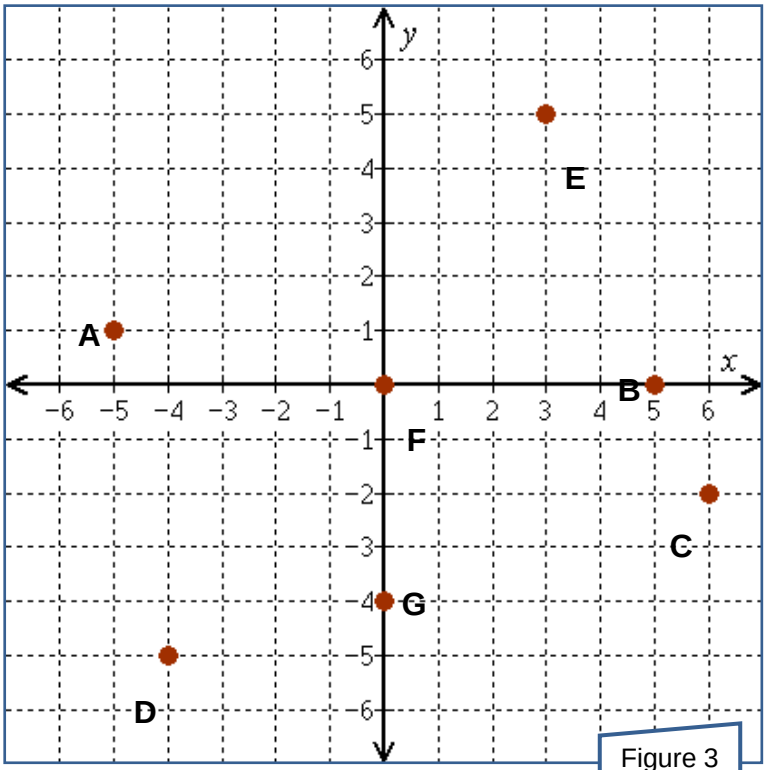

Figure 3

Example 7

Plot the points $\mathbf{A}(-3,4)$, $\mathbf{B}(-3,-3)$, and $\mathbf{C}(5,2)$ onto Figure 4 and find the area, A, of the resultant triangle. Assume that the scale on each axis is in centimeters.

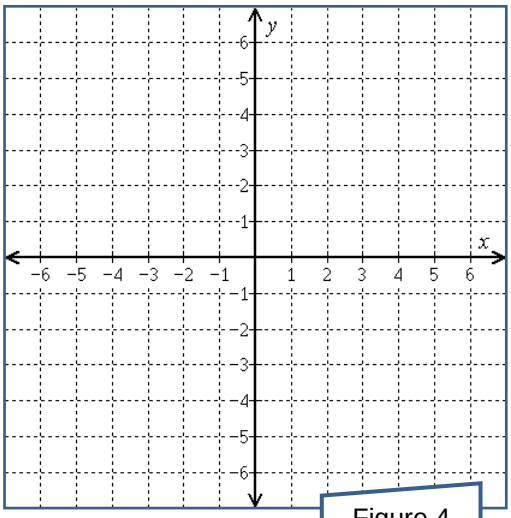

Figure 4

Example 8

At each point on the line in Figure 5, twice the x-coordinate minus the y-coordinate is always the same number. What is this common difference?

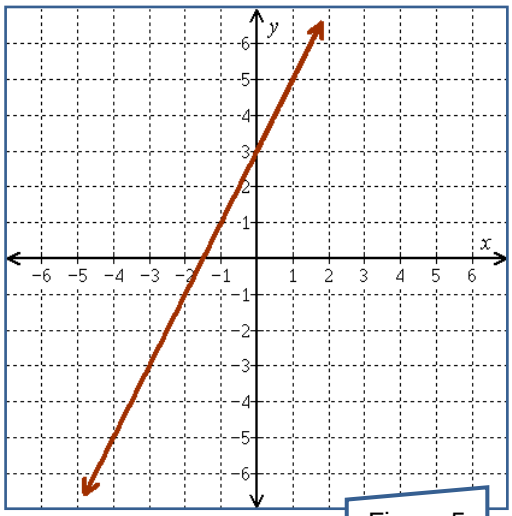

Figure 5

Table 5

point	ordered pair	$2x - y$

Example 9

Complete Table 6 with solutions to the equation $y = -\frac{3}{2}x$. Then graph the solutions and show that they are collinear.

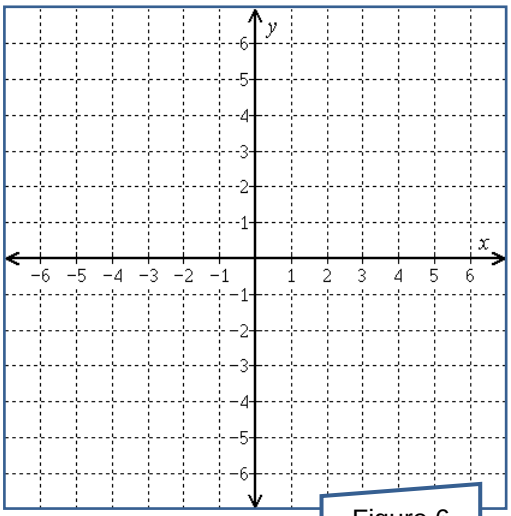

Figure 6

Table 6: $y = -\frac{3}{2}x$

x	y
-4	
0	
2	

Definition

An equation that *can* be written in the form $Ax + By = C$ (not both A and B zero) is called a ***linear equation*** of x and y. The graph of all of the solutions to a linear equation with two variables is a (straight) line (when graphed in the rectangular coordinate plane).

Three or more points in the plane are ***collinear*** (lie on a common line) if and only if they all satisfy a common linear equation.

Example 10

Complete Table 7 with four solutions to the equation $x + 2y = 6$. Then graph the solutions and show that they are collinear.

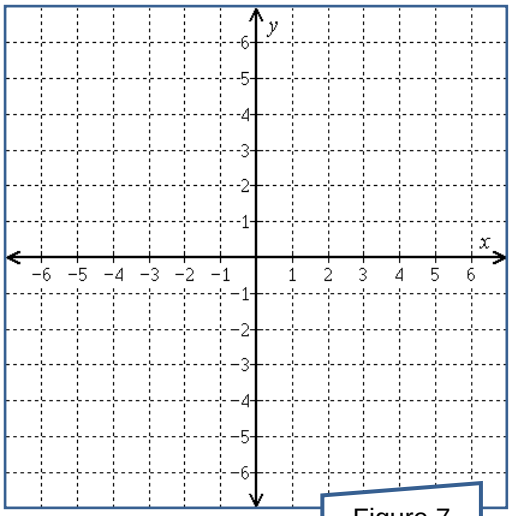

Figure 7

Table 7: $x + 2y = 6$

x	y

Example 11

Complete Table 8 with four solutions to the equation $3x - 2y = 6$. Then graph the solutions and show that they are collinear.

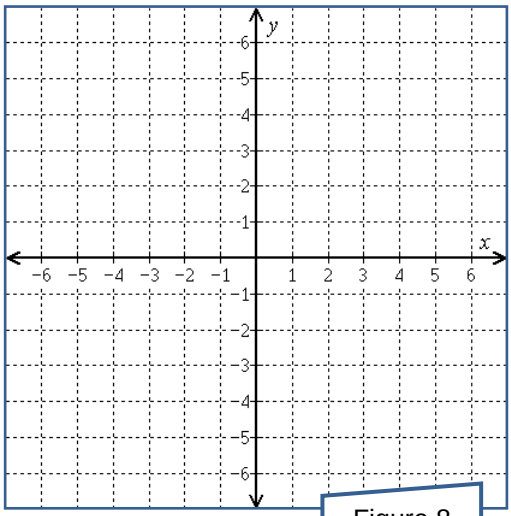

Figure 8

Table 8: $3x - 2y = 6$

x	y

Definition

When a line or a curve is drawn in the xy-plane, any point on the line or curve that also lies on the y-axis is called a ***y-intercept*** and any point on the line or curve that also lies on the x-axis is called an ***x-intercept*** .

x-intercept: $(x, 0)$ ***y-intercept***: $(0, y)$

Example 12

State all of the intercepts of the curve shown in Figure 9.

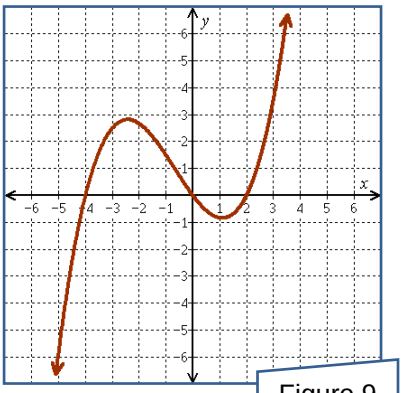

Figure 9

Example 13

Find the intercepts of the line with equation $3x - 5y = -20$.

Example 14

Plot the line $2x + 4y = -8$ onto Figure 10 after first finding the intercepts of the line. Find a third point on your plotted line and show that it also satisfies the equation.

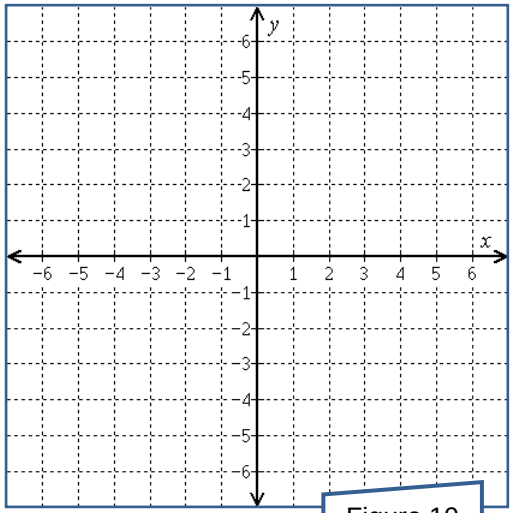

Figure 10

Example 15

Plot onto Figure 11 several points in the xy-plane that satisfy the equation $x = -4$. What do you observe? What are the intercepts of the resultant curve?

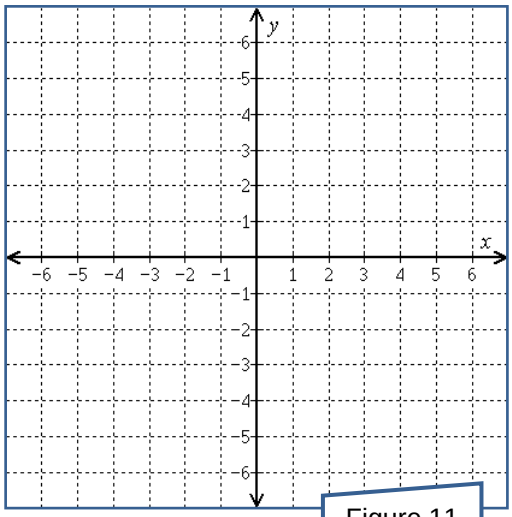

Figure 11

Table 11: $x = -4$

x	y

Example 16

What is an equation for the line in Figure 12? What are the intercepts of the line?

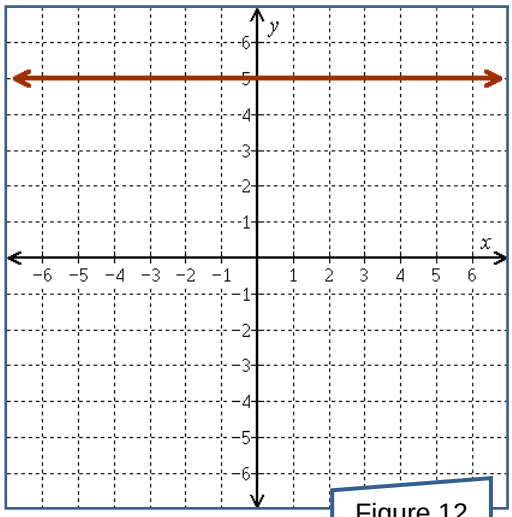

Figure 12