Definitions

A solution to an equation with two variables, x and y, is an ***ordered pair*** (a,b) where a and b are real numbers with the property that if $x=a$ and $y=b$ the equation is true. The number a is called the ***x-coordinate*** of the ordered pair and the number b is called the ***y-coordinate*** of the ordered pair.

Example 1

What are the x and y coordinates of the ordered pair $(9,-2)$. Is the ordered pair a solution to the equation $3x+4y=10+x$?

The x-coordinate is 9 and the y-coordinate is -2.

$3(9)+4(-2) \stackrel{?}{=} 10+9$

$27-8 = 19$ oh yeah!

So $(9,-2)$ is $\underline{\underline{\underline{\text{a}}}}$ solution to $3x+4y=10+x$

Example 2

What is the ordered pair with an x-coordinate of -3 that satisfies the equation $y=\frac{1}{2}x-1$

when $x=-3$, $y=\frac{1}{2}x-1$

$= \frac{1}{2}(-3)-1$

$= -5/2$

The ordered pair is $(-3, -5/2)$.

Example 3

What is the ordered pair with a y-coordinate of -3 that satisfies the equation $y=\frac{1}{2}x-1$

When $y=-3$, $-3 = \frac{1}{2}x-1$

$-3+1 = \frac{1}{2}x-1+1$

$-2 = \frac{1}{2}x$

$2(-2) = 2(\frac{1}{2}x)$

$-4 = x$

The ordered pair is $(-4,-3)$.

Example 3

Complete Table 1.

Table 1: $y = -3x - 4$

x	y
2	-10
-2	2

$$\underline{x = 2}$$
$$y = -3(2) - 4$$
$$= -10$$

$$\underline{y = 2}$$
$$2 = -3x - 4$$
$$2 + 4 = -3x - 4 + 4$$
$$6 = -3x$$
$$\frac{6}{-3} = \frac{-3x}{-3}$$
$$-2 = x$$

Example 4

Write the missing values into Table 2 so that each implied ordered pair is a solution to the equation $x + y = 4$. Plot the four ordered pairs onto Figure 1.

Table 2: $x + y = 4$

x	y
0	4
4	0
6	-2
2	2
–1.5	5.5

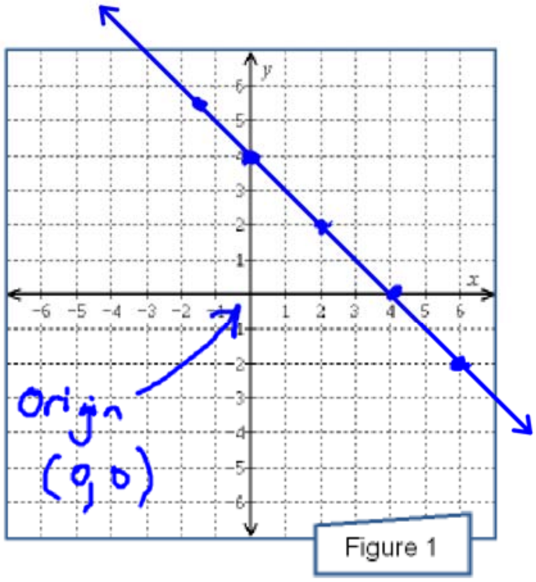
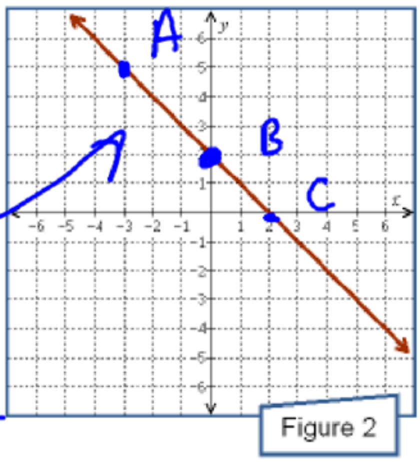


Figure 1

Example 5

The sum of the the coordinates of each point on the line in Figure 2 is always the same number. What is this constant sum?



This is the line $x+y=2$

Figure 2

y-intercept

x-intercept

Table 3

point	ordered pair	$x + y$
A	(-3, 5)	2
B	(0, 2)	2
C	(2, 0)	2

Example 6

Several points are shown in Figure 3. State the ordered pair associated with each point and where in the coordinate plane the point lies; assume that both coordinates of each point are intergers. Which point, B or F, is a solution to the equation $3x - 2y = 8$?

Table 4

Point	Coordinates	Location
A	(-5, 1)	Quad II
B	(5, 0)	x-axis
C	(6, -2)	Quad IV
D	(-4, -5)	Quad III
E	(3, 5)	Quad I
F	(0, 0)	the origin!
G	(0, -4)	y-axis

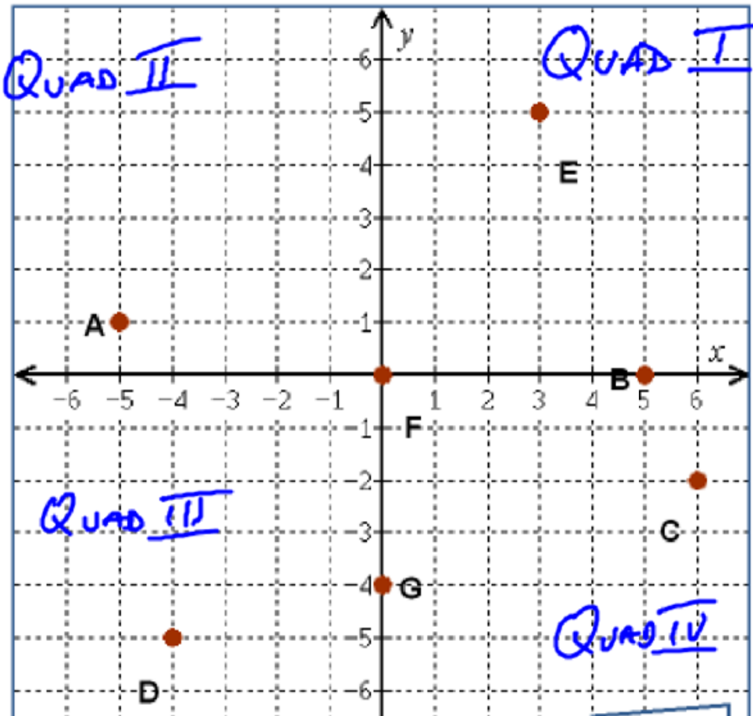


Figure 3

The axes break the coordinate plane into quadrants.

every pt is either

1) in a quadrant
2) on one-axis
3) the origin

$\text{Area} = \frac{1}{2}bh$

Example 7

Plot the points $\mathbf{A}(-3,4)$, $\mathbf{B}(-3,-3)$, and $\mathbf{C}(5,2)$ onto Figure 4 and find the area, A, of the resultant triangle. *Assume that the scale on each axis is in centimeters.*

$b = 7\text{ cm}$

$h = 8\text{ cm}$

The area is

$\frac{1}{2}bh = \frac{1}{2}(7\text{cm})(8\text{cm})$

$= 28\text{ cm}^2$

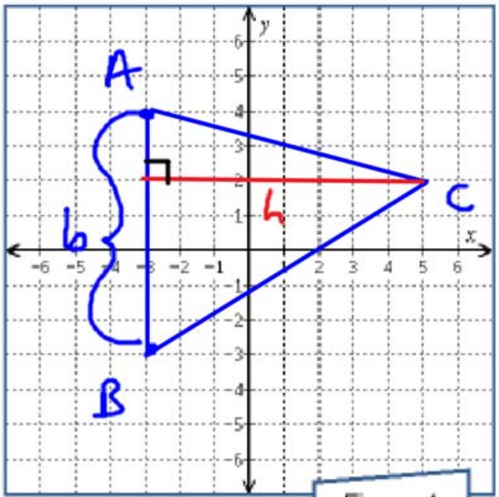


Figure 4

Example 8

At each point on the line in Figure 5, twice the x-coordinate minus the y-coordinate is always the same number. What is this common difference?

This is the line $2x - y = -3$

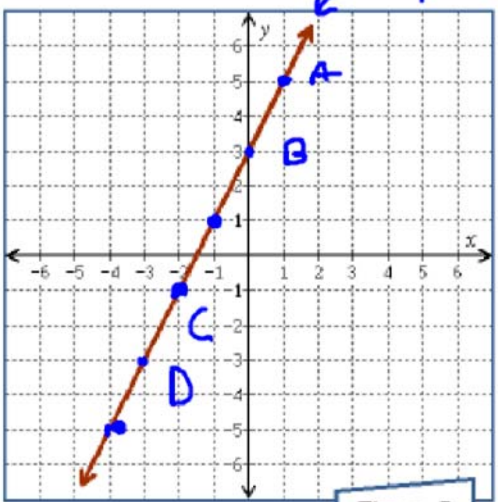


Figure 5

Table 5

point	ordered pair	$2x-y$
A	$(1,5)$	-3
B	$(0,3)$	-3
C	$(-2,-1)$	-3
D	$(-3,-3)$	-3

Example 9

Complete Table 6 with solutions to the equation $y = -\frac{3}{2}x$. Then graph the solutions and show that they are collinear.

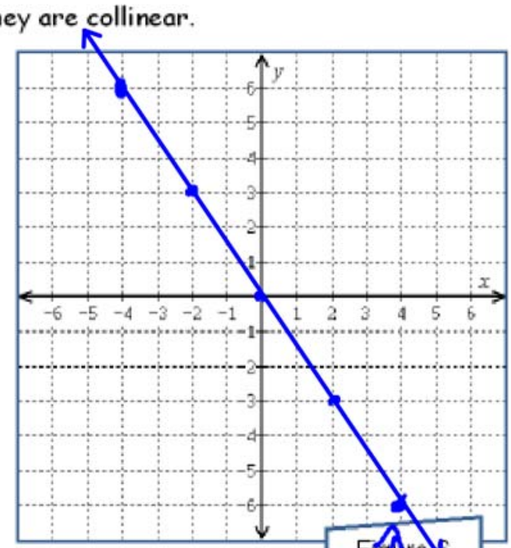


Figure 6

(4, -6)

Table 6: $y = -\frac{3}{2}x$

x	y
-4	6
0	0
2	-3

$-\frac{3}{2}(4) = \frac{-12}{2} = -6$ ✓

> **Definition**
>
> An equation that *can* be written in the form $Ax + By = C$ (not both A and B zero) is called a ***linear equation*** of x and y. The graph of all of the solutions to a linear equation with two variables is a (straight) line (when graphed in the rectangular coordinate plane).
>
> Three or more points in the plane are ***collinear*** (lie on a common line) if and only if they all satisfy a common linear equation.

$y = \#x + b$

Example 10

Complete Table 7 with four solutions to the equation $x + 2y = 6$. Then graph the solutions and show that they are collinear.

Check

$-6 + 2(6) = 6$?

$-6 + 12 = 6$ ✓

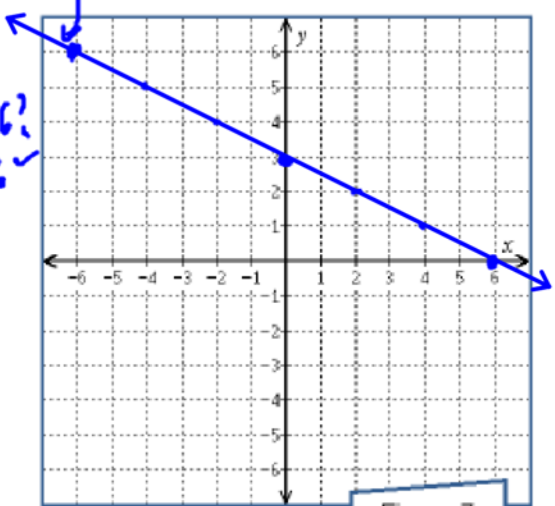


Figure 7

y-intercept

x-intercept

Table 7: $x + 2y = 6$

x	y
0	3
6	0
-6	6
2	2

$2 + 2(2) = 6$ ✓

Example 11

Complete Table 8 with four solutions to the equation $3x-2y=6$. Then graph the solutions and show that they are collinear.

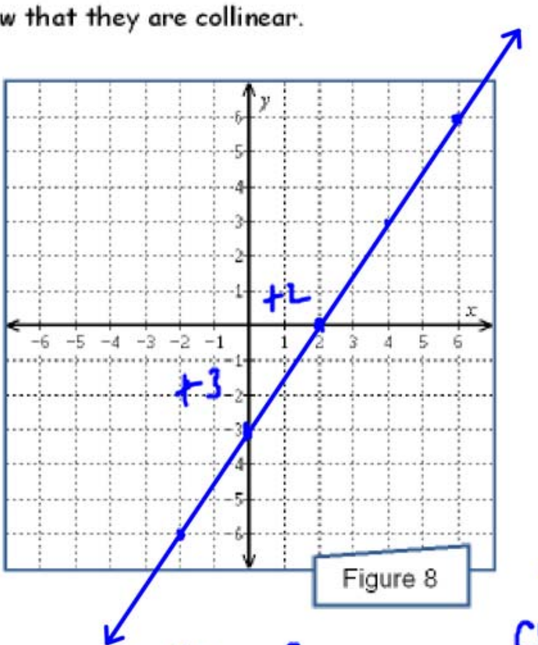


Figure 8

Table 8: $3x-2y=6$

x	y
0	−3
2	0
4	3
−2	−6

Slope is $\frac{\Delta y}{\Delta x} = \frac{3}{2}$

Check

$3(4)-2(3)=6?$

$12-6=6$ ✓

Check

$3(-2)-2(-6)=6?$

$-6+12=6$ ✓

Definition

When a line or a curve is drawn in the xy-plane, any point on the line or curve that also lies on the y-axis is called a ***y*-intercept** and any point on the line or curve that also lies on the x-axis is called an ***x*-intercept**.

***x*-intercept**: $(x,0)$ ***y*-intercept**: $(0,y)$

Example 12

State all of the intercepts of the curve shown in Figure 9.

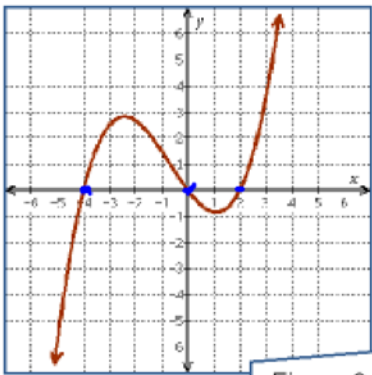
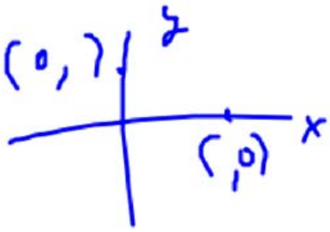


Figure 9

The x-intercepts are $(-4,0)$, $(0,0)$, and $(2,0)$.

The y-intercept is $(0,0)$.



Example 13

Find the intercepts of the line with equation $3x - 5y = -20$.

x-Intercept

$y = 0$

$3x = -20$

$\frac{3x}{3} = \frac{-20}{3}$

$x = -20/3$

y-intercept

$x = 0$

$-5y = -20$

$\frac{-5y}{-5} = \frac{-20}{-5}$

$y = 4$

The x-intercept is $(-20/3, 0)$ and the y-intercept is $(0, 4)$.

Example 14

Plot the line $2x + 4y = -8$ onto Figure 10 after first finding the intercepts of the line. Find a third point on your plotted line and show that it also satisfies the equation.

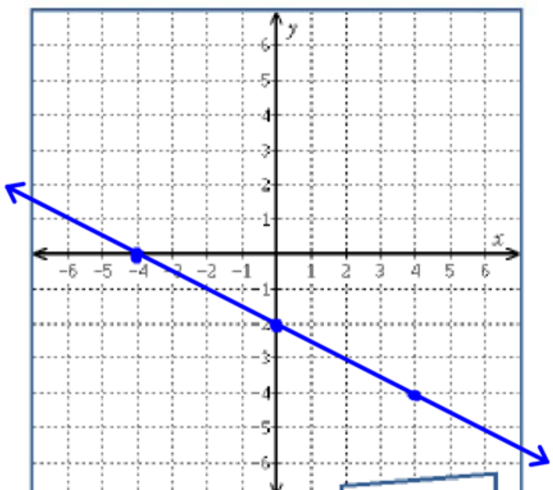


Figure 10

x-intercept: $(-4, 0)$

y-intercept: $(0, -2)$

Check $(4, -4)$

$2(4) + 4(-4) = -8$?

$8 + (-16) = -8$ ✓

Example 15

Plot onto Figure 11 several points in the xy-plane that satisfy the equation $x = -4$. What do you observe? What are the intercepts of the resultant curve?

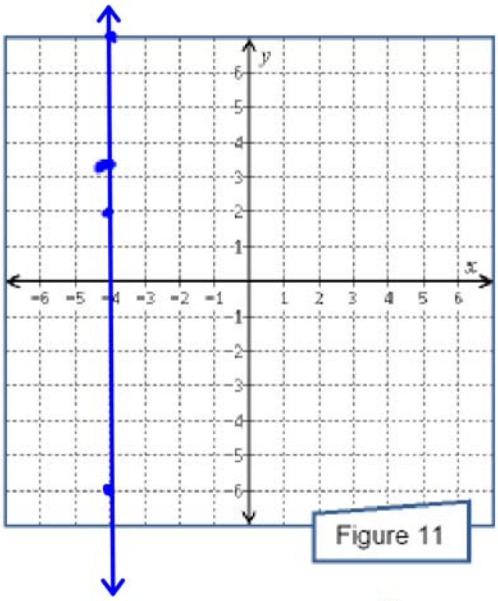


Figure 11

Table 11: $x = -4$

x	y	
-4	7	x=-4? ✓
-4	2	x=-4? ✓
-4	-6	x=-4? ✓
-4	3.1642	x=-4? ✓

There ain't no y-intercept.

The x-intercept is (-4, 0).

By Jove, x = k is always a vertical line (k is a number)

Example 16

What is an equation for the line in Figure 12? What are the intercepts of the line?

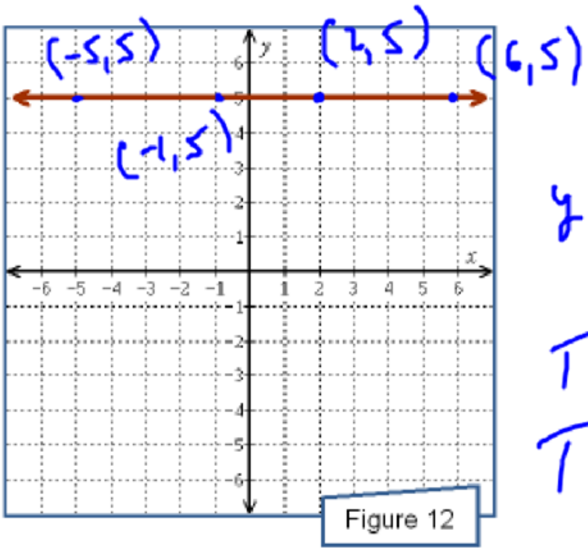


Figure 12

y = 5

y = k is always a horizontal line.

The y-intercept is (0, 5)

There ain't no x-intercept.