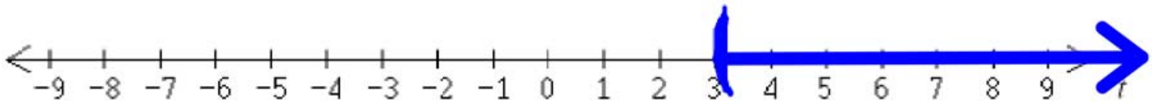
Example 1

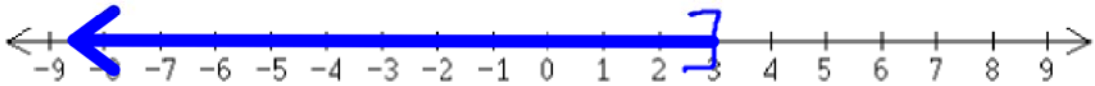
Provide the missing information for each inequality.

inequality: $t > 3$



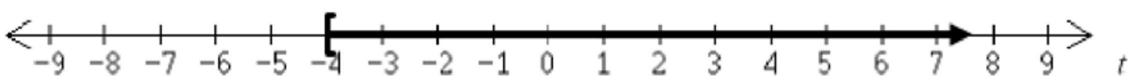
set builder notation	interval notation	interval type
$\{t \mid t > 3\}$	$(3, \infty)$	open

inequality: $t \leq 3$



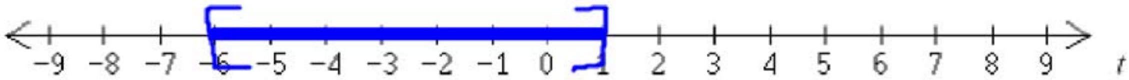
set builder notation	interval notation	interval type
$\{t \mid t \leq 3\}$	$(-\infty, 3]$	half open

inequality: $t \geq -4$



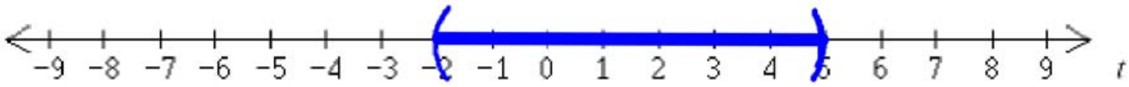
set builder notation	interval notation	interval type
$\{t \mid t \geq -4\}$	$[-4, \infty)$	half open

inequality: $-6 \le t \le 1$



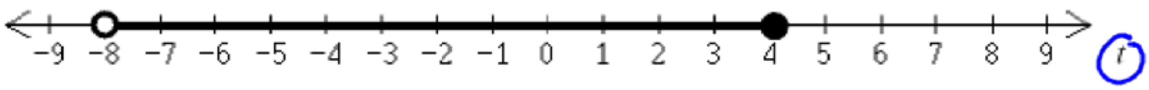
set builder notation	interval notation	interval type
$\{t \mid -6 \le t \le 1\}$	$[-6, 1]$	closed

inequality: $-2 < t < 5$



set builder notation	interval notation	interval type
$\{t \mid -2 < t < 5\}$	$(-2, 5)$	open

inequality: $-8 < t \le 4$



set builder notation	interval notation	interval type
$\{t \mid -8 < t \le 4\}$	$(-8, 4]$	half open

Solution set

The addition property of inequalities

If $a < b$, then $a + c < b + c$ If $a \le b$, then $a + c \le b + c$

If $a > b$, then $a + c > b + c$ If $a \ge b$, then $a + c \ge b + c$

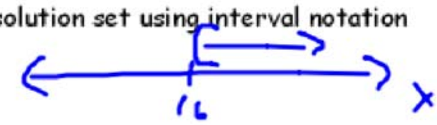
Example 2

Find the solution set to the inequality $x - 7 \ge 9$. State the solution set using interval notation

$x - 7 \ge 9$

$x - 7 + 7 \ge 9 + 7$

$x \ge 16$



The solution set to $x - 7 \ge 9$ is $[16, \infty)$.

Example 3

Find the solution set to inequality $x + 7 \ge 94$. State the solution set using set builder notation.

$x + 7 \ge 94$

$x + 7 - 7 \ge 94 - 7$

$x \ge 87$

The solution set to $x + 7 \ge 94$ is $\{x \mid x \ge 87\}$

Example 4

Find the solution set to inequality $-25 \le w - 15 < 90$. Graph the solution set.

$-25 \le w - 15$ $\underline{\underline{\text{and}}}$ $w - 15 < 90$

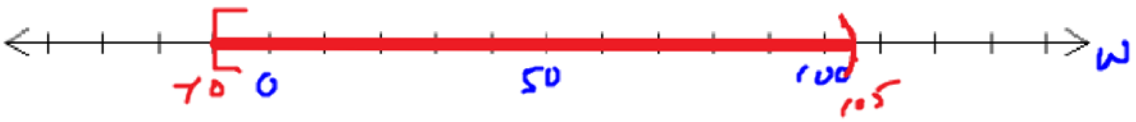
$-25 + 15 \le w - 15 + 15$ $\underline{\underline{\text{and}}}$ $w - 15 + 15 < 90 + 15$

CUT TO THE CHASE...

$-25 \le w - 15 < 90$

$-25 + 15 \le w - 15 + 15 < 90 + 15$

$-10 \le w < 105$



The scale is $\underline{\underline{10}}$ (distance between ticks)

Example 5

Insert the proper inequality symbol (less than or greater than) between each pair of numbers.

Before sense: $<$

After sense: $<$

$3 < 5$

$2(3) < 2(5)$

$6 < 10$

Before sense: $>$

After sense: $>$

$-6 > -12$

$\frac{-6}{3} > \frac{-12}{3}$

$-2 > -4$

Before sense: $<$

After sense: $>$

$0 < 2$

$-5(0) > -5(2)$

$0 > -10$

Before sense: $>$

After sense: $<$

$8 > -4$

$\frac{8}{-4} < \frac{-4}{-4}$

$-2 < 1$

Observation

The sense reverses when you multiply or divide by a negative number.

The sense remains the same when you multiply or divide by a positive number.

A strategy for solving linear inequities of form $ax + b < cx + d$

(Note that the strategy directly adapts to the other three inequality signs.)

1. Use the addition property of inequalities to isolate the variable term *on the left side* of the inequality sign and the constant term *on the right side* of the inequality sign.

2. Apply the multiplication property of inequalities to completely isolate the variable *on the left side* of the inequality sign. ***Always*** explicitly show this step!

   Remember – When you **multiply** both sides of the inequality **by a positive number** the inequality sign maintains its **original direction** (sense).

   Remember – When you **multiply** both sides of the inequality **by a negative number** the inequality sign **reverses direction** (sense).

3. Make sure that you state your solution set in the requested format(s).

Example 6

Find the solution set to the inequality $4x - 7 \geq 2x + 9$. State the solution set using interval notation.

$$4x - 7 \geq 2x + 9$$

$$4x - 7 - 2x \geq 2x + 9 - 2x$$

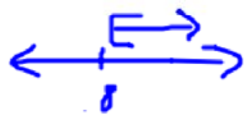
$$2x - 7 \geq 9$$

$$2x - 7 + 7 \geq 9 + 7$$

$$2x \geq 16$$

$$\frac{2x}{2} \geq \frac{16}{2}$$

$$x \geq 8$$



The solution set is $[8, \infty)$.

Example 7

Find the solution set to the inequality $3t + 15 > 9(t + 1)$. State the solution set using interval notation.

$$3t + 15 > 9(t+1)$$

$$3t + 15 > 9t + 9$$

$$3t + 15 - 9t > 9t + 9 - 9t$$

$$-6t + 15 > 9$$

$$-6t + 15 - 15 > 9$$

$$-6t > -6$$

$$\frac{-6t}{-6} < \frac{-6}{-6}$$

$$t < 1$$

The solution set is $(-\infty, 1)$

Checking is 2 steps
1) The endpoint always makes the two sides equal

$4-(3(0)-2) \overset{?}{=} 2+2(2-0)$
$4-(-2) \overset{?}{=} 2+4$ ✓

Test # for inequality
$x \le 0$: test -1
$4-(-5) \ge 2+2(3)$?
$9 \ge 8$ ✓

Example 8

Find the solution set to the inequality $4-(3x-2) \ge 2+2(2-x)$. State the solution set using set builder notation.

$4-(3x-2) \ge 2+2(2-x)$

$4-3x+2 \ge 2+4-2x$

$-3x+6 \ge -2x+6$

$-3x+6+2x \ge -2x+6+2x$

$-x+6 \ge 6$

$-x+6-6 \ge 6-6$

$-x \ge 0$

$\frac{-x}{-1} \le \frac{0}{-1}$

$x \le 0$

The solution set is

$\{x \mid x \le 0\}$.

Example 9

J. Z.'s HPE teacher likes to keep grading simple. The teacher assigns grades based upon attendance (out of 100 points), a midterm and final exam - the two tests are each also worth 100 points. J.Z. was a bad boy and only earned 65 of his attendance points. The lack of attendance was reflected in J.Z.'s midterm score, which was 62. J.Z. needs 210 points total to scrape a C out of the class. What is the minimum score J.Z. can earn on his final to scrape his C? Use a proper inequality to determine your solution.

Let x represent J.Z.'s score on the final.

$65+62+x \ge 210$

$x+127 \ge 210$

$x+127-127 \ge 210-127$

$x \ge 83$

J.Z. needs at least an 83 to squeak his C.