More equation Solving

Find and state the solution to each equation.

a. $4x + 7 = 27$

b. $3x - 6 = 3x - 6$

c. $7x + 2 = 9x + 2$

d. $4 - x = 8 - x$

a. $4x+7=27$

$4x+7-7=27-7$

$4x = 20$

$\frac{4x}{4} = \frac{20}{4}$

$x = 5$

The solution set to $4x+7=27$ is $\{5\}$.

b. $3x-6=3x-6$

$3x-6-3x=3x-6-3x$

$-6=-6$

This is called an identity.

[no matter what the value of x, $3x-6$ equals $3x-6$!!]

Every # is a solution.

The solution set is $\mathbb{R}$ ↑ all real numbers

c. $7x+2=9x+2$

$7x+2-7x=9x+2-7x$

$2=2x+2$

$2-2=2x+2-2$

$0=2x$

$\frac{0}{2}=\frac{2x}{2}$

$0=x$

The solution set is $\{0\}$.

d. $4-x=8-x$

$4-x+x=8-x+x$

$4=8$

This is called a contradiction

The equation $4-x=8-x$ has no solutions.

The solution set is

Stop!! Do not turn the page!!

$\{\}$ ↑ empty set → $\emptyset$

$\{\emptyset\}$

The solution to a linear equation in one variable always has one of three forms.

- There is exactly one solution to the equation. If the solution to the equation is the number ***k***, then ***the solution set is*** $\{k\}$.
- There are **no** solutions to the equation. These type equations are called ***contradictions***. We call ***the solution set*** to these equations ***the empty set*** **(written as** $\varnothing$ **or** $\{\}$ **)**.
- Every real number is a solution to the equation. These type equations are called ***identities***. We call ***the solution set*** to these equations ***all real numbers*** **(written as** $\mathbb{R}$ **)**.

Recognizing identities and contradictions

- An equation that is equivalent to an equation of form $k = k$ where k is a real number is itself an identity,
- An equation that is equivalent to an equation of form $a = b$ where a are b are two *different* numbers is itself a contradiction,

Be careful!!!

Don't confuse your solution process with your check. When *checking* your solution, arriving at $k = k$ simply means that your solution is correct – not that every real number satisfies the original equation!

BOO BOO™

Be careful!!!

If you have found a "solution" and your solution doesn't check – it does **not** mean that the equation has no solution!! It means that you've got to go find where you made a mistake!

e. $5x-(x-7)=1-2x+2(x+13)$

$5x-(x-7)=1-2x+2(x+13)$

$5x-x+7=1-2x+2x+26$

$4x+7=27$

$4x+7-7=27-7$

$4x=20$

$\frac{4x}{4}=\frac{20}{4}$

$x=5$

Check

$5(5)-(5-7)=1-2(5)+2(5+13)$?

$25-(-2)=1-10+2(18)$?

$27=-9+36$?

$27=27$ ✓

The solution set is $\{5\}$.

f. $4x+3\left(x+\frac{2}{3}\right)=9x+2$

$4x+3(x+\frac{2}{3})=9x+2$

$9x+3x+2=9x+2$

$7x+2=9x+2$

$7x+2-7x=9x+2-7x$

$2=2x+2$

$2-2=2x+2-2$

$0=2x$

$\frac{0}{2}=\frac{2x}{2}$

$0=x$

The solution is 0.

The solution set is $\{0\}$.

$3(x+\frac{2}{3})=3x+\frac{1}{1}\cdot\frac{2}{1}$

$=3x+2$

$\frac{1}{4} \cdot \frac{3}{5} = \frac{1 \cdot 3}{4 \cdot 1} = 3/4$

$\frac{1}{4} \cdot \frac{24}{5}$

e. $\frac{3}{4}x - \frac{3}{2} = \frac{5}{4}\left(\frac{3}{5}x - \frac{24}{5}\right)$

$\frac{3}{4}x - \frac{3}{2} = \frac{5}{4}\left(\frac{3}{5}x - \frac{24}{5}\right)$

$\frac{3}{4}x - \frac{3}{2} = \frac{3}{4}x - 6$

$\frac{3}{4}x - \frac{3}{2} - \frac{3}{4}x = \frac{3}{4}x - 6 - \frac{3}{4}x$

$-\frac{3}{2} = -6$ contradiction

The solution set is $\{\}$.

f. $6 - (x + 2) = -(x - 4)$

$6 - 1(x + 2) = -1(x - 4)$

$6 - x - 2 = -x + 4$

$4 - x = -x + 4$

$4 - x + x = -x + 4 + x$

$4 = 4$ word! It's an identity!

The solution set is $\mathbb{R}$.