

Definition

A **linear equation in one variable**, x , is an equation that can be written in the form

$$ax + b = c \text{ where } a, b, \text{ and } c \text{ are real numbers, } a \neq 0.$$

Example 1

For each equation either show that it is a linear equation in one variable (and state the variable) or state why the equation is not a linear equation.

a. $-6 - 8t = 4$

$$-8t + (-6) = 4 \quad \text{This is linear with variable } t.$$

b. $x = 9$

$$1 \cdot x + 0 = 9 \quad \text{This is linear with variable of } x.$$

c. $6x^2 - 8 = 6$

This is not linear, x is squared.

d. $14 = 14$

$0x + 14 = 14$ but this equation is not linear because $a \neq 0$.

Definition

Two equations are called **equivalent equations** if they have exactly the same solution(s).

$$\underbrace{x+7=17 \quad t-6=4}_{\text{equivalent}}$$

Example 2

Decide whether or not each pair of equations is equivalent and explain your reasoning.

a. $x+7=17$ and $x-6=4$.

These are equivalent equations; the have the same solution (10).

b. $t+3=23$ and $-t=20$.

These are not equivalent.
20 solves $t+3=23$
-20 solves $-t=20$

c. $9-y=9$ and $8+y^2=8$

These equations are equivalent (solution is 0).

The Addition Property of Equations

Adding the same number or algebraic expression to both sides of an equation results in an equivalent equation.

NOTE: In later classes you will learn that there are exceptions to this rule – you do not need to worry about this now, however. For the types of equations you are solving in this class the statement is true 100% of the time.

Example 3

Find the solution to each equation after first using the addition property of equations to find an equivalent equation of form $x=k$ or $k=x$ where k is a real number. Focus on the process! We are establishing building blocks upon which the remainder of your algebra career will in part be based.

$$x-9=41$$

Ask yourself: What can I add to $x-9$ so that I'm just left with x ?

$$\begin{array}{l|l} x-9=41 & \text{The solution} \\ x-9+9=41+9 & \text{to } x-9=41 \\ x=50 & \text{is 50.} \end{array}$$

$$\frac{5}{2} + \left(-\frac{21}{2}\right) = x + \frac{21}{2} + \left(-\frac{21}{2}\right)$$

$$\frac{5}{2} = x + \frac{21}{2}$$

Ask yourself: What can I subtract from $x + \frac{21}{2}$ so that I'm just left with x ?

$$\frac{5}{2} = x + \frac{21}{2}$$

$$\frac{5}{2} - \frac{21}{2} = x + \frac{21}{2} - \frac{21}{2}$$

$$\frac{-16}{2} = x$$

$$-8 = x$$

The solution

to

$$\frac{5}{2} = x + \frac{21}{2}$$

is -8 .

$$4x = 5 + 3x$$

Ask yourself: What can I subtract from $5 + 3x$ so that I'm just left with 5 ?

$$4x = 5 + 3x$$

$$4x - 3x = 5 + 3x - 3x$$

$$x = 5$$

The solution

to $4x = 5 + 3x$

is 5

check

$$4(5) = 5 + 3(5)?$$

$$20 = 5 + 15 \checkmark$$

$$6x + 18 = -5 + 7x$$

Ask yourself: What can I subtract from $6x + 18$ so that I'm just left with 18 ?

Ask yourself: What can I add to $-5 + x$ so that I'm just left with x ?

$$6x + 18 = -5 + 7x$$

$$6x + 18 - 6x = -5 + 7x - 6x$$

$$18 = -5 + x$$

$$18 + 5 = -5 + x + 5$$

$$23 = x$$

The solution

to $6x + 18 = -5 + 7x$

is 23 .

check

$$6(23) + 18 = -5 + 7(23)?$$

$$138 + 18 = -5 + 161?$$

$$156 = 156 \checkmark$$

The Multiplication Property of Equations

Multiplying both sides of an equation by the same *non-zero number* results in an equivalent equation.

NOTE: This property is also true if you divide both sides of an equation by the same *non-zero number*.

Example 4

Find the solution to each equation after first using the multiplication property of equations to find an equivalent equation of form $x = k$ or $k = x$ where k is a real number. Focus on the process! We are establishing building blocks upon which the remainder of your algebra career will in part be based.

$$\frac{1}{5}x = 35$$

Ask yourself: What can I multiply $\frac{1}{5}x$ by so that I'm just left with x ?

$$\begin{aligned}\frac{1}{5}x &= 35 \\ 5\left(\frac{1}{5}x\right) &= 5(35) \\ x &= 175\end{aligned}$$

The solution to $\frac{1}{5}x = 35$ is 175.

$$8 = 4x$$

Ask yourself: What can I multiply $4x$ by so that I'm just left with x ?

$$\begin{aligned}8 &= 4x \\ \frac{1}{4}(8) &= \frac{1}{4}(4x) \\ 2 &= x\end{aligned}$$

The solution to $8 = 4x$ is 2.

$$8 = 4x$$

Ask yourself: What can I divide $4x$ by so that I'm just left with x ?

$$\begin{aligned}8 &= 4x \\ \frac{8}{4} &= \frac{4x}{4} \\ 2 &= x\end{aligned}$$

The solution to $8 = 4x$ is 2.

$$-x = -7$$

Ask yourself: What can I multiply $-x$ by so that I'm just left with x ?

$$\begin{array}{l} -x = -7 \\ -1(-x) = -1(-7) \\ x = 7 \end{array} \quad \left| \quad \begin{array}{l} \text{The solution to} \\ -x = -7 \text{ is} \\ 7. \end{array} \right.$$

Solving Linear Equations in One Variable (x)

1. **Completely simplify** the expressions on both sides of the equal sign.
2. Use the addition property of equations once or twice, as necessary, so that you have an equivalent equation of form $ax = b$ or $b = ax$ where a and b are real numbers.
3. Use the multiplication property of equations once, as necessary, so that you have an equivalent equation of form $x = k$ or $k = x$ where k is a real number.
4. Check your solution **in the original equation**. If your solution doesn't check, **find and fix your mistake**.
5. State your solution using a complete English sentence.

Example 5

Solve each equation. Focus on the process! We are establishing building blocks upon which the remainder of your algebra career will in part be based.

$$8x - 19 = -35$$

$$\begin{array}{l} 8x - 19 = -35 \\ 8x - 19 + 19 = -35 + 19 \\ 8x = -16 \\ \frac{8x}{8} = \frac{-16}{8} \\ x = -2 \end{array}$$

$$\begin{array}{l} \text{Check} \\ 8(-2) - 19 = -35? \\ -16 - 19 = -35 \\ \checkmark \end{array}$$

The solution to
 $8x - 19 = -35$
is -2 .

$$14 = 12 + 4y$$

$$\begin{aligned} 14 &= 12 + 4y \\ 14 - 12 &= 12 + 4y - 12 \\ 2 &= 4y \\ \frac{1}{4}\left(\frac{2}{1}\right) &= \frac{1}{4}(4y) \\ \frac{1}{2} &= y \end{aligned}$$

The solution to
 $14 = 12 + 4y$
is $\frac{1}{2}$.

$$7w - 15 = 4w$$

$$\begin{aligned} 7w - 15 &= 4w \\ 7w - 15 - 4w &= 4w - 4w \\ 3w - 15 &= 0 \\ 3w - 15 + 15 &= 0 + 15 \\ 3w &= 15 \end{aligned}$$

$$\begin{aligned} \frac{3w}{3} &= \frac{15}{3} \\ w &= 5 \end{aligned}$$

The solution
to $7w - 15 = 4w$
is 5.

$$7w - 15 = 4w$$

$$\begin{aligned} 7w - 15 &= 4w \\ 7w - 15 - 7w &= 4w - 7w \\ -15 &= -3w \\ \frac{-15}{-3} &= \frac{-3w}{-3} \\ 5 &= w \end{aligned}$$

The solution to the
equation is 5.

$$3 - 5x = -27 - 20x$$

$$3 - 5x = -27 - 20x$$

$$3 - 5x + 20x = -27 - 20x + 20x$$

$$3 + 15x = -27$$

$$3 + 15x - 3 = -27 - 3$$

$$15x = -30$$

$$\frac{15x}{15} = \frac{-30}{15}$$

$$x = -2$$

Check

$$3 - 5(-2) = -27 - 20(-2)?$$

$$3 + 10 = -27 + 40?$$

$$13 = 13 \checkmark$$

The solution to $3 - 5x = -27 - 20x$ is -2 .

$$6t - 2(t - 4) = -(t - 3)$$

$$6t - 2(t - 4) = -(t - 3)$$

$$6t - 2t + 8 = -t + 3$$

$$4t + 8 = -t + 3$$

$$4t + 8 + t = -t + 3 + t$$

$$5t + 8 = 3$$

$$5t + 8 - 8 = 3 - 8$$

$$5t = -5$$

$$\frac{5t}{5} = \frac{-5}{5}$$

$$t = -1$$

Check

$$6(-1) - 2(-1 - 4) = -(-1 - 3)?$$

$$-6 - 2(-5) = -(-4)?$$

$$-6 + 10 = 4 \checkmark$$

The solution to $6t - 2(t - 4) = -(t - 3)$ is -1 .

$$5x + 22 - 15x = 10(x + 1)$$

$$5x + 22 - 15x = 10(x + 1)$$

$$-10x + 22 = 10x + 10$$

$$-10x + 22 + 10x = 10x + 10 + 10x$$

$$22 = 20x + 10$$

$$22 - 10 = 20x + 10 - 10$$

$$12 = 20x$$

$$\frac{12}{20} = \frac{20x}{20}$$

$$\frac{3}{5} = x \text{ (yee!)}$$

Check

$$5\left(\frac{3}{5}\right) + 22 - 15\left(\frac{3}{5}\right) \stackrel{?}{=} 10\left(\frac{3}{5} + 1\right)$$

$$3 + 22 - 9 \stackrel{?}{=} 10\left(\frac{8}{5}\right)$$

$$16 = 16$$

The solution to $5x + 22 - 15x = 10(x + 1)$ is $\frac{3}{5}$.

Practice Questions

Work all of these problems on a separate sheet of paper showing steps in a way that you think will earn you 100% credit when you take the test. The only way you will know if you are showing work the way Mr. Simonds wants to see it is if you show that work and let Mr. Simonds take a look at it.

1. Solve each equation. Remember ... your goal is not simply to figure out "the answer." You also need to practice showing the steps using appropriate formatting. You can write out the steps in an appropriate manner, but it will take practice before it becomes habit.

a. $3x + 12 = 48$

b. $\frac{1}{2}x - 6 = -10$

c. $4t + 19 = 7t - 6$

d. $-x + 19 = -31$

e. $4(2y - 3) = -12$

f. $2w + \frac{12}{5} = \frac{1}{10} - 2w$