Multiplication and Division of Signed Numbers

The product or quotient of two numbers with the same sign is positive.
The product or quotient of two numbers with opposite signs is negative.

Example 1

Find each product or quotient.

$(-9)\cdot 8 =$ -72 $\qquad (-9)\cdot(-8) =$ 72 $\qquad 9\cdot(-8) =$ -72

$\dfrac{(-6)(-4)}{(-2)(-3)} = \dfrac{24}{6} = 4$ $\quad ((-6)(-4)) \div ((-2)(-3))$

$\dfrac{(-6)(4)}{(-2)(-3)} = \dfrac{-24}{6} = -4$

$(-3)(-1)(-6)(-2)(-3) = (3)(-6)(-2)(-3)$
$= (-18)(-2)(-3)$
$= (36)(-3)$
$= -108$
$=$

$(3)(-1)(6)(2)(-3) = (-3)(6)(2)(-3)$
$= (-18)(2)(-3)$
$= (-36)(-3)$
$= 108$
$=$

When the number of negative factors in the product or quotient is even, the simplified product or quotient is positive.

When the number of negative factors in the product or quotient is odd, the simplified product or quotient is negative.

Example 2

Find each of the following and state what you observe.

$\frac{-8}{4} =$ -2 $\qquad \frac{8}{-4} =$ -2 $\qquad -\frac{8}{4} =$ -2

negative fractions have the easiest strike tone

negative sign anywhere $\left\{ \frac{8}{4} \right.$

Order of Operations (aka PEMDAS)

> Remember, if you break the rules of Order of Operations you'll be darn lucky if you come up with the right value!
>
> **PEMDAS**
>
> **P**arentheses (and other grouping symbols)
> **E**xponents
> **M**ultiplication/**D**ivision (left to right if there are successive occurrences)
> **A**ddition/**S**ubtraction (left to right if there are successive occurrences)

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$12 \times 8 = 96$ $\qquad 12 = \frac{96}{8}$

Example 3

Evaluate the expression $\frac{2}{5} + 3\left(x^3 + 2y\right)$ when $x = \frac{1}{2}$ and $y = 6$.

when $x = \frac{1}{2}$ and $y = 6$

$$\frac{2}{5} + 3(x^3 + 2y) = \frac{2}{5} + 3\left(\left(\tfrac{1}{2}\right)^3 + 2(6)\right)$$

$$= \frac{2}{5} + 3\left(\frac{1}{8} + 12\right)$$

$$= \frac{2}{5} + \frac{3}{1}\left(\frac{97}{8}\right)$$

$$= \frac{2}{5} + \frac{291}{8}$$

$$= \frac{2(8) + 291(5)}{5(8)}$$

$$= \frac{16 + 1455}{40}$$

$$12 = \frac{12}{1} \times \frac{8}{8} = \frac{96}{8}$$

$$\frac{2}{5} \times \frac{8}{8} + \frac{291}{8} \times \frac{5}{5}$$

$$\frac{2(8) + 291(5)}{40}$$

$$= \frac{1471}{40}$$

Example 4

Evaluate the expression $-\frac{4}{9} \div \left(x - y + \frac{1}{4} \right)$ when $x = \frac{1}{6}$ and $y = \frac{1}{3}$.

When $x = \frac{1}{6}$ and $y = \frac{1}{3}$,

Make sure that your scratch work is boxed off from your other work.

$$-\frac{4}{9} \div \left(\frac{1}{6} - \frac{1}{3} + \frac{1}{4} \right) = -\frac{4}{9} \div \left(\frac{2}{12} - \frac{4}{12} + \frac{3}{12} \right)$$

$$= -\frac{4}{9} \div \left(-\frac{2}{12} + \frac{3}{12} \right)$$

$$= -\frac{4}{9} \div \frac{1}{12}$$

$$= -\frac{4}{9} \times \frac{12}{1}$$

$$= -\frac{16}{3}$$

$$\boxed{\frac{-4}{\cancel{9}_{3}} \times \frac{\cancel{12}^{4}}{1}}$$

Example 5

Evaluate the expression $\frac{xy}{x - 2z}$ when $x = 10$, $y = 8$, and $z = 3$.

When $x = 10$, $y = 8$, and $z = 3$

$$\frac{xy}{x - 2z} = \frac{(10)(8)}{10 - 2(3)}$$

$$= \frac{80}{10 - 6}$$

$$= \frac{80}{4}$$

$$= 20$$

A fraction bar acts like a grouping symbol when you follow PEMDAS. In this example,

$$\frac{xy}{x - 2z} = (xy) \div (x - 2z)$$

$8 \div 2 \times 4$

P/E /MD/AS

Mr. Simonds' MTH 60 class

Division and multiplication are done left to right

Example 6

Evaluate the expression $n \div 3 \cdot n$ when $n = 9$.

When $n = 9$

$$n \div 3 \cdot n = 9 \div 3 \cdot 9$$
$$= 3(9)$$
$$= 27$$

$$\frac{9}{3}(9)$$

Example 7

Evaluate the expressions $3(x+y)$ and $3x+3y$ when $x=2$ and $y=5$. What happens? Is this a surprise?

When $x=2$ and $y=5$

$$3(x+y) = 3(2+5)$$
$$= 3(7)$$
$$= 21$$

$$3x+3y = 3(2)+3(5)$$
$$= 6+15$$
$$= 21$$

OF COURSE I GOT THE SAME NUMBER. DISTRIBUTIVE PROPERTY! WORD, MAN!

Example 8

Find each of the following and state what you observe.

$-1\cdot(-2) =$ 2 $\qquad -(-2) =$ 2

the opposite of negative 2

negative 1 times negative 2

$-1\cdot(2) =$ -2 $\qquad -(2) =$ -2

the opposite of 2

negative 1 times 2

Multiplying by negative one is the same as taking the opposite of the number.

An important new fact related to order of operations

Unless a negative sign is in parentheses, exponents come before negation in order of operations.

Example 9

Find each of the following. Write phrases that describe each original expression.

$(-3)^2 = \underline{(-3)(-3)} = \underline{9}$

"negative three" squared

$2+(3\cdot 4)$

$-1\cdot 3^2 = \underline{-1\cdot 9} = \underline{-9} = \underline{\quad}$

$-(3^2)$

$-3^2 = \underline{-1\cdot 3^2} = \underline{-1\cdot 9} = \underline{-9}$

the opposite .... of three squared

$x^2 = x\cdot x$
if $x=-7$
we need $(-7)(-7)$

Example 10

Evaluate $-7^2 + x^2$ when $x = -7$.

When $x=-7$,

$-7^2 + x^2 = -7^2 + (-7)^2$

"negative 7" square

$= -49 + 49$

0

$$2 - (x+4) = 2 + (-1)(x+4)$$
$$= 2 + [(-1)x + (-1)4]$$
$$= 2 + (-x + -4)$$
$$= 2 - x - 4$$

$$2 - 7 = 2 + (-7)$$
$$= 2 + (-1)(7)$$

Example 11

Completely simplify each expression.

Simplified expressions contain no grouping symbols

All like terms are combined in simplified expressions.

$3+7\left[2-(x+4)\right]$

$$3+7[2-(x+4)] = 3+7[2-x-4]$$
$$= 3+7[-x-2]$$
$$= 3+(-7x)-14$$
$$= -7x-11$$

$4x^2-3\left(7x-12x^2\right)+19x$

$$4x^2-3(7x-12x^2)+19x = 4x^2-21x+36x^2+19x$$
$$= 40x^2-2x$$

Example 12

Bono decided to change careers and needed to take an algebra class to get into his program of choice. Bono was taking a test and started to doubt himself about whether or not he could combine $5x^2$ and $4x^3$. Specifically, Bono was thinking that maybe the answer was $9x^5$. What's a way Bono could have used numbers to see whether or not $5x^2+4x^3=9x^5$?

Let $x=2$, then

$$5x^2+4x^3 = 5(2^2)+4(2^3)$$
$$= 5(4)+4(8)$$
$$= 20+32$$
$$= 52$$

but ... $9x^5 = 9(2^5)$
$= 9(32)$
$\neq 52!$

$5x^2+4x^3 \neq 9x^5!$

$2(x+y) = 2x+2y$
$-1(x+y) = (-1)(x) + (-1)(y)$

Example 13

Distribute each subtraction sign *but do not combine like terms.* What do you observe?

$4-(2x+7)$

$4-(2x+7) = 4-2x-7$

$4-(-2x-7)$

$4-(-2x-7)$
$= 4+2x+7$

$4-(2x-7)$

$4-(2x-7) = 4-2x+7$

$4-(-2x+7)$

$4-(-2x+7) = 4+2x-7$

Example 14

Distribute each negative sign. What do you observe?

$-\left(x^3+6x\right)$

$-(x^3+6x) = -x^3-6x$

$-\left(-x^3-6x\right)$

$-(-x^3-6x) = x^3+6x$

$-\left(x^3-6x\right)$

$-(x^3-6x) = -x^3+6x$

$-\left(-x^3+6x\right)$

$-(-x^3+6x) = x^3-6x$

Example 15

Completely simplify $3-\left[5x^2-\left(-3x-9+4x^2\right)\right]$.

$$3-[5x^2-(-3x-9+4x^2)] = 3-[5x^2+3x+9-4x]$$
$$= 3-[5x^2-x+9]$$
$$= 3-5x^2+x-9$$
$$= -5x^2+x-6$$

Example 16

Consider the expression $2(x+y)-\left(-x^2+2y\right)$

a. Evaluate the expression when $x=-3$ and $y=5$.

When $x=-3$ and $y=5$

$$2(x+y)-(-x^2+2y) = 2(-3+5)-(-(-3)^2+2(5))$$
$$= 2(2)-(-9+10)$$
$$= 4-1$$
$$= 3$$

b. Simplify the expression,

$$2(x+y)-(-x^2+2y) = 2x+2y+x^2-2y$$
$$= 2x+x^2$$

c. Check your simplification by evaluating it when $x=-3$ and $y=5$.

When $x=-3$ and $y=5$

$$2x+x^2 = 2(-3)+(-3)^2$$
$$= -6+9$$
$$= 3$$