The Associative Property of Addition

If a, b, and c are any real numbers, then $a+(b+c)=(a+b)+c$.

The Associative Property of Multiplication

If a, b, and c are any real numbers, then $a(bc)=(ab)c$.

Example 1

Simplify each of the following expressions after applying the associative property of addition or the associative property of multiplication.

a. $3+(9+x)$

$$3+(9+x)=(3+9)+x$$
$$=12+x$$

b. $(t+8)+22$

$$(t+8)+22=t+(8+22)$$
$$=t+30$$

c. $3(25y)$

$$3(25y)=(3\cdot 25)y$$
$$=75y$$

d. $(5x)x$

$$(5x)x=5(xx)$$
$$=5x^2$$

The Distributive Properties

Multiplication distributes over addition	**Multiplication distributes over subtraction**
If a, b, and c are any real numbers, then:	If a, b, and c are any real numbers, then:
$a(b+c) = ab + ac$	$a(b-c) = ab - ac$
and	and
$(b+c)a = ba + ca$	$(b-c)a = ba - ca$

Example 2

Apply the distributive property to each expression and simplify the result.

a. $3(x+4)$

$$3(x+4) = 3x + 3(4)$$
$$= 3x + 12$$

b. $(x-2)y$

$$(x-2)y = xy - 2y$$

c. $5(3t-4)$

$$5(3t-4) = 5(3t) - 5(4) \quad \text{Distributi}$$
$$= (5 \cdot 3)t - 20 \quad \text{Associate}$$
$$= 15t - 20 \quad \text{simplify}$$

d. $(u+v)7$

$$(u+v)7 = 7u + 7v$$

Example 3

Use the distributive property to combine the terms.

a. $3x + 11x$

$$3x + 11x = (3 + 11)x$$
$$= 14x$$

b. $6t + 10.4t$

$$6t + 10.4t = (6 + 10.4)t$$
$$= 16.4t$$

$$\frac{57 \div 3}{12 \div 3} \quad \frac{19}{4}$$

c. $\frac{5}{4}y + \frac{21}{6}y$

$$\frac{5}{4}y + \frac{21}{6}y = \left(\frac{5}{4} + \frac{21}{6}\right)y$$
$$= \left(\frac{5}{4}\cdot\frac{3}{3} + \frac{21}{6}\cdot\frac{2}{2}\right)y$$
$$= \frac{15 + 42}{12}y$$
$$= \frac{57}{12}y$$
$$= \frac{19}{4}y$$

d. $9w + \frac{29}{5}w$

$$9w + \frac{29}{5}w = \left(9 + \frac{29}{5}\right)w$$
$$= \left(\frac{9}{1}\cdot\frac{5}{5} + \frac{29}{5}\right)w$$
$$= \left(\frac{45}{5} + \frac{29}{5}\right)w$$
$$= \frac{45 + 29}{5}w$$
$$= \frac{74}{5}w$$

Some definitions and thoughts

The ***terms*** of an expression are the expressions that are added together.

The ***factors*** of a term are the expressions that are multiplied together.

Two terms are called ***like terms*** or ***common terms*** if:

- They both contain no variables. (These are called ***constant terms***.)
- They both contain exactly the same variables and the common variables share the same exponent.

Terms of form "number times variable" (e.g. $7x$ or $16a$) are called ***linear terms***.

Regardless of the form of the variable factor, the numeric factor of a term is called the ***coefficient*** of the term.

Example 4

For the following expression identify each of the following:

- The terms as well as their coefficients.
- The common terms.
- The linear terms and constant terms.

$$5x + 9 + 7 + 4y + x + 10 + xy + 3y^2$$

Terms / coefficients

$5x$ / 5
9 / 9
7 / 7
$4y$ / 4
x / 1
10 / 10
xy / 1
$3y^2$ / 3

Common terms

$5x, x$
$9, 7, 10$

constants

$9, 7, 10$

linear

$5x, 4y, x$

Some useful facts and ideas

Two terms of an expression can be combined if and only if they are common terms. When ***simplifying an expression*** we always combine common terms.

When combining like terms we are technically applying the distributive property. In essence, however, we are really only counting like objects which, in effect, comes down to adding or subtracting the coefficients of the common terms.

Example 5

Completely simplify each expression. You do not need to show any steps unless there is distribution that needs to be done.

a. $8x+9+12x-4$

$$8x+9+12x-4=(8x+12x)+(9-4)$$
$$=20x+5$$

b. $2x+(4y+8x)$

$$2x+(4y+8x)=10x+4y$$

c. $3+2(t+9)$

multiply first

$$3+2(t+9)=3+2t+2(9)$$
$$=3+2t+18$$
$$=2t+21$$

d. $\frac{5}{4}(8+3x)+\frac{1}{2}(6x+9y+1)$

$$\frac{5}{4}(8+3x)+\frac{1}{2}(6x+9y+1)=\frac{5}{4}\cdot\frac{8}{1}+\frac{5}{4}\cdot\frac{3x}{1}+\frac{1}{2}\cdot\frac{6x}{1}+\frac{1}{2}\cdot\frac{9y}{1}+\frac{1}{2}\cdot 1$$
$$=\frac{40}{4}+\frac{15x}{4}+\frac{6x}{2}+\frac{9y}{2}+\frac{1}{2}$$
$$=\frac{40}{4}+\frac{15x}{4}+\frac{12x}{4}+\frac{18y}{4}+\frac{2}{4}$$
$$=\frac{15x+12x}{4}+\frac{18y}{4}+\frac{40+2}{4}$$
$$=\frac{27x}{4}+\frac{18y}{4}+\frac{42}{4}$$
$$=\frac{27}{4}x+\frac{9}{2}y+\frac{21}{2}$$

fi si

The Commutative Property of Addition

If a and b are any real numbers, then $a + b = b + a$.

The Commutative Property of Multiplication

If a and b are any real numbers, then $a\,b = b\,a$.

Example 6

Although I would *never* expect you to show all of the following steps, it is comforting to know that there are rules of algebra that justify the things we do when simplifying algebraic expressions. For each line with a blank, state the property of real numbers that was applied on that line..

$4 + 6(x+7) = 4 + (6x + 42)$ Distributed the 6

$= 4 + (42 + 6x)$ Commuted 42 & 6x

$= (4 + 42) + 6x$ associative property of addition

$= 46 + 6x$ added 4+42 (simplified)

$= 6x + 46$ Commutative property

$x + (9x + 8) = (x + 9x) + 8$ associative property

$= (1+9)x + 8$ distributive property backwards

$= 10x + 8$ added 1+9 (simplified)