Definition

A solution to an inequality in 2 variables is an ordered pair that makes the inequality true.

Example 1

Is $(9,-2)$ a solution to the inequality $2x+3y \geq 5$?

$2(9)+3(-2) \geq 5$?

$18-6 \geq 5$ True

$(9,-2)$ is a solution.

Example 2

Is $(6,4)$ a solution to the inequality $x<5$?

$6<5$ no!

$(6,4)$ is not a solution.

Example 3

Is $(0,1)$ a solution to the inequality $y \leq 4x+1$?

$1 \leq 4(0)+1$?

$1 \leq 1$ True

$(0,1)$ is a solution.

Example 4

Is $(3,-2)$ a solution to the inequality $4x+6y<0$?

$4(3)+6(-2)<0$?

$0<0$? No!

$(3,-2)$ isn't a solution

Fact

Other than a couple of exceptional cases, the solution set to a linear inequality in 2 variables is a half plane. Graphs are the most common way we communicate solution sets to linear inequalities in 2 variables.

Strategy 1 for graphing the solution set to a linear inequality in 2 variables

Begin by isolating y ***on the left side*** of the inequality symbol.

- If the inequality has form $y \geq mx + b$ the solution set is all points ***on or above the line*** $y = mx + b$. Draw the line $y = mx + b$ and shade above the line.
- If the inequality has form $y > mx + b$ the solution set is all points ***above the line*** $y = mx + b$. Draw a ***dotted*** line $y = mx + b$ and shade above the line.
- If the inequality has form $y \leq mx + b$ the solution set is all points ***on or below the line*** $y = mx + b$. Draw the line $y = mx + b$ and shade below the line.
- If the inequality has form $y < mx + b$ the solution set is all points ***below the line*** $y = mx + b$. Draw a ***dotted*** line $y = mx + b$ and shade below the line.

Example 5

Graph onto Figure 1 the solution set to the inequality $y \geq \frac{1}{2}x - 2$.

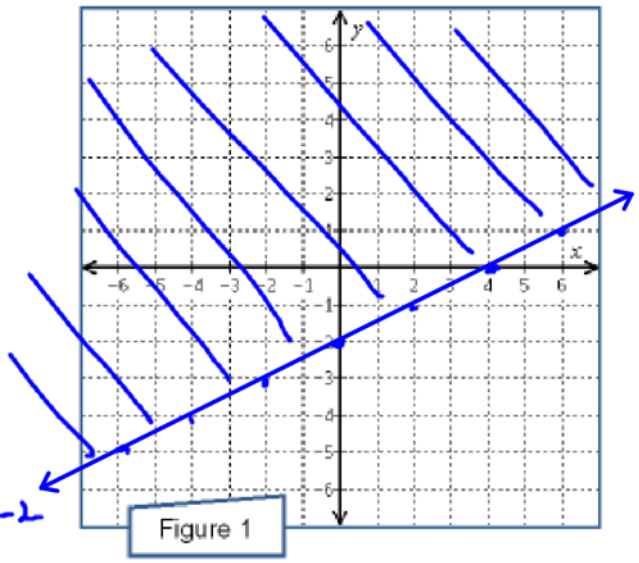


Figure 1

(1) Graph $y = \frac{1}{2}x - 2$ solid

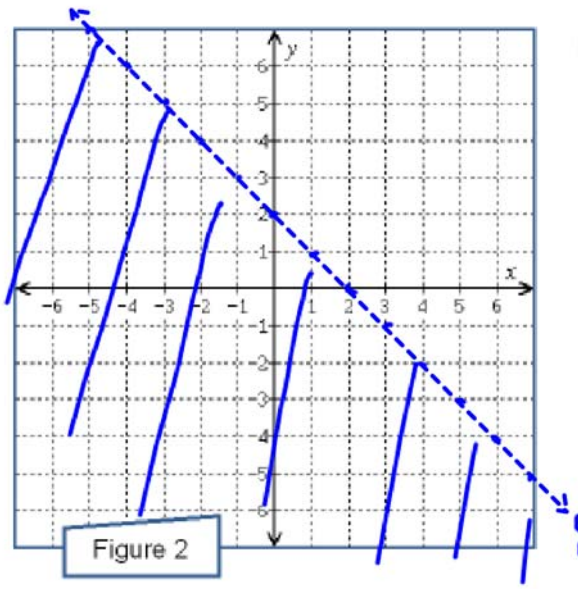
check (4, 0)

$0 = \frac{1}{2}(4) - 2$ True

$y = \frac{1}{2}x - 2$

Example 6

Graph onto Figure 2 the solution set to the inequality $y < 2 - x$.



① Graph $y = 2 - x$ Dashed

$y = -x + 2$

y-int $(0, 2)$ $m = -1$

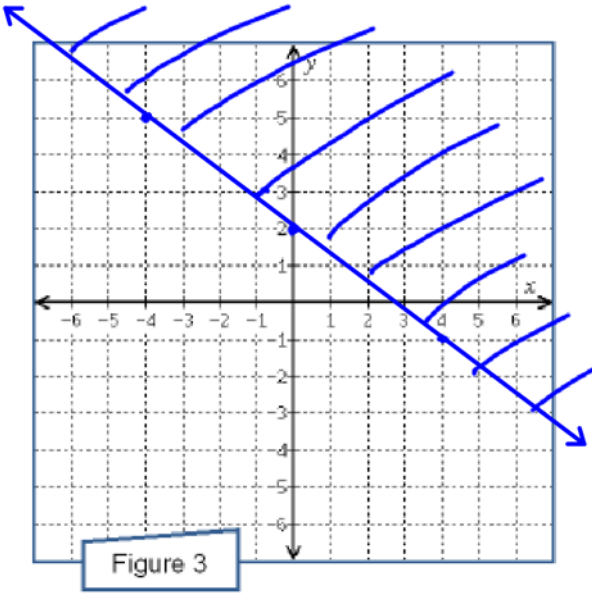
Check $(3, -1)$

$-1 = 2 - 3$ True dat!

$y = 2 - x$

Example 7

Graph onto Figure 3 the solution set to the inequality $3x + 4y \geq 8$.



$3x + 4y \geq 8$

$3x + 4y - 3x \geq 8 - 3x$

$4y \geq -3x + 8$

$\frac{4y}{4} \geq \frac{-3x+8}{4}$

$y \geq -\frac{3}{4}x + 2$

① Graph $y = -\frac{3}{4}x + 2$ solid

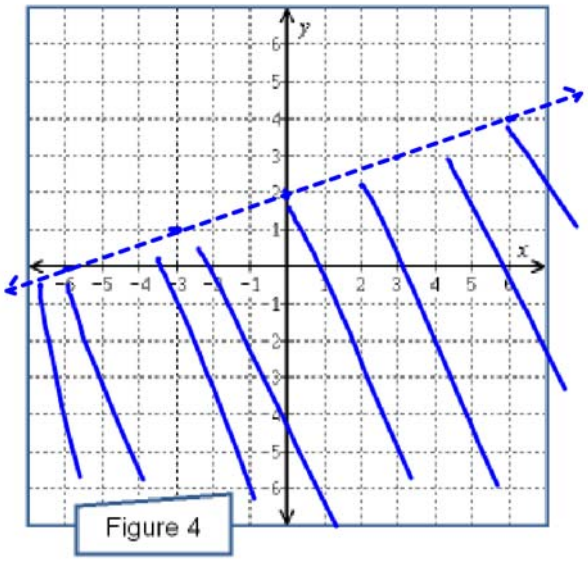
Check $(4, -1)$ in $3x + 4y = 8$

$3(4) + 4(-1) = 8$?

$12 - 4 = 8$ ✓

Example 8

Graph onto Figure 4 the solution set to the inequality $x - 3y > -6$.



$x - 3y > -6$

$x - 3y - x > -6 - x$

$-3y > -x - 6$

$\frac{-3y}{-3} < \frac{-x-6}{-3}$

$y < \frac{1}{3}x + 2$ dashed

Graph $y = \frac{1}{3}x + 2$

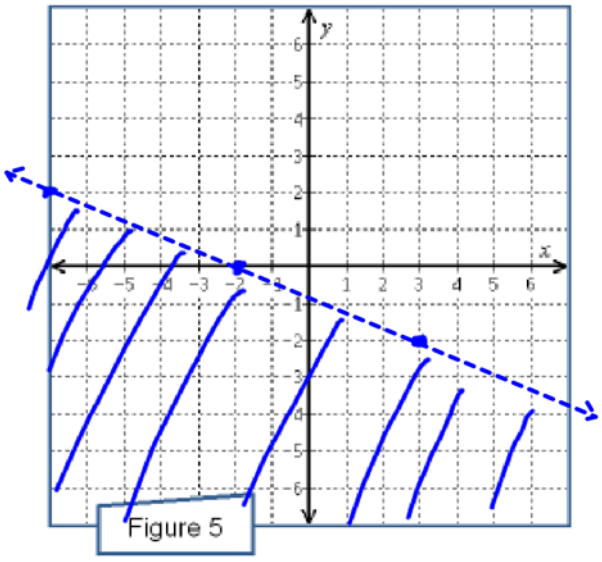
check (3, 1)

$1 = \frac{1}{3}(3) + 2$?

$1 = 1 + 2$ ✓

Example 9

Graph onto Figure 5 the solution set to the inequality $-2x - 5y > 4$.



$-2x - 5y > 4$

$-2x - 5y + 2x > 4 + 2x$

$-5y > 2x + 4$

$\frac{-5y}{-5} < \frac{2x+4}{-5}$

$y < -\frac{2}{5}x - \frac{4}{5}$

① Graph $y = -\frac{2}{5}x - \frac{4}{5}$ dashed

Think

$y = \frac{-2x-4}{5}$

(3, −2)

(−2, 0) ✓

(−7, 2) ✓

Look for a "multiple of −2" −4 that is evenly divisible by 5

Strategy 2 for graphing the solution set to a linear inequality in 2 variables

Begin by graphing the boundary line that results from replacing the inequality sign with an equal sign. Remember to use a solid line if the inequality symbol is $\leq$ or $\geq$ and a dotted line if the inequality symbol is $<$ or $>$.

Next, test a point ***not on the line*** in ***the inequality***.

- If the point makes the inequality ***true*** shade ***that side*** of the line.
- If the point makes the inequality ***false*** shade ***the other side*** of the line.

Example 10

Graph onto Figure 6 the solution set to the inequality $4x + 3y \geq 12$.

① Graph $4x+3y=12$ Solid

x	y
3	0
0	4

check (6, -4)

$4(6)+3(-4)=12$?

$24-12=12$ ✓

② Test a point not on the line in original inequality

Use, eg, (0, 0)

$4(0)+3(0) \geq 12$ not!

Shade the other side of the line

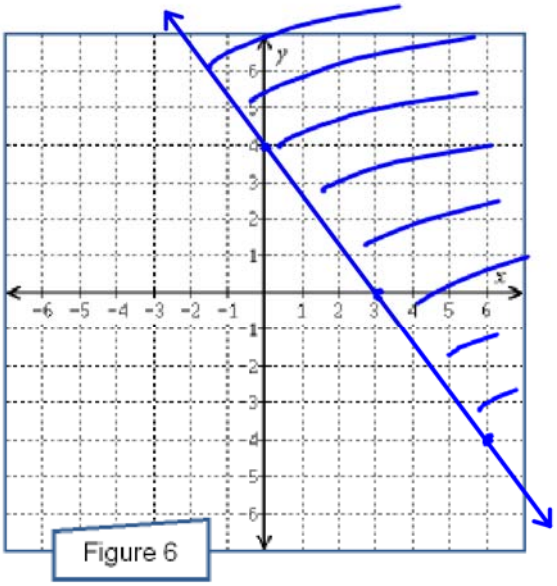


Figure 6

Example 11

Graph onto Figure 7 the solution set to the inequality $x \geq 3$.

① Graph $x=3$

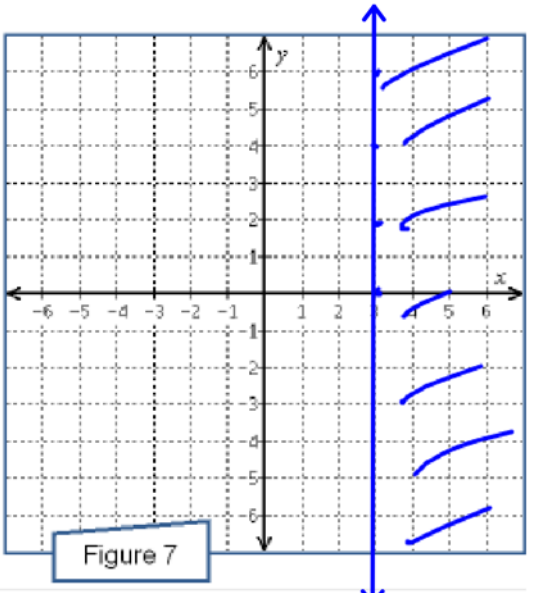
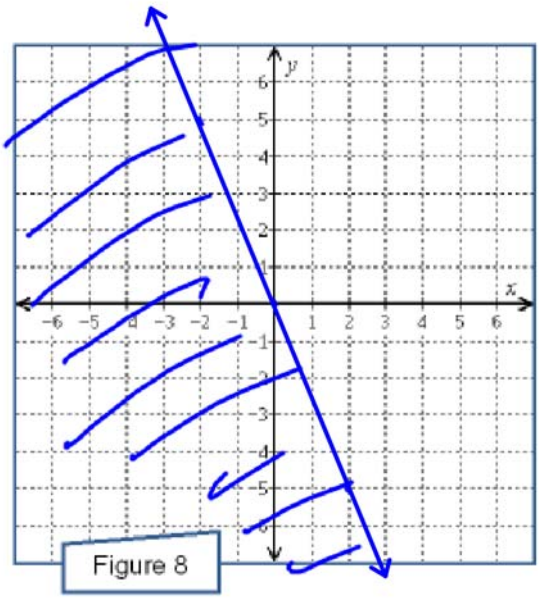


Figure 7

10 - 10 = 0

Example 12

Graph onto Figure 8 the solution set to the inequality $-5x - 2y \geq 0$.



(1) Graph $-5x - 2y = 0$

x	y
0	0
-2	5

Test (2, -5)

$-5(2) - 2(-5) = 0$?

$-10 + 10 = 0$?

True!

(2) Test (1, 1) in the inequality

$-5(1) - 2(1) \geq 0$?

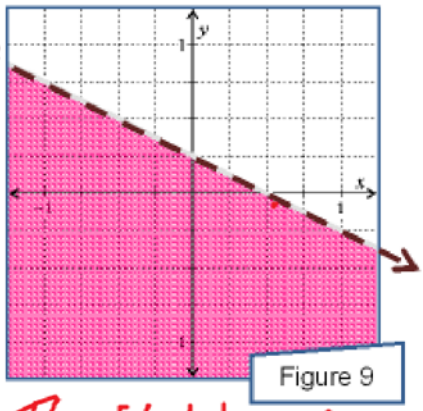
$-7 \geq 0$ But!

(1, 1) is on the loser side

Example 13

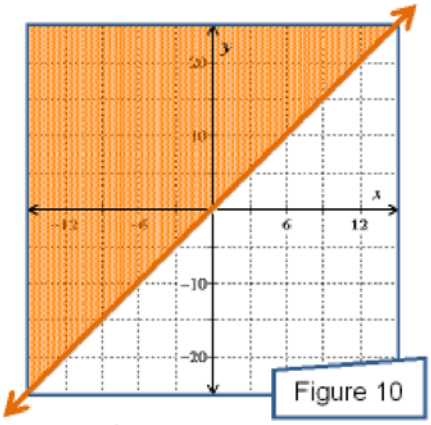
Find inequalities that would graph to the sets indicated in figures 9 and 10.

This line is $y = -\frac{1}{2}x + \frac{1}{4}$



The shaded region is the solution set to $y < -\frac{1}{2}x + \frac{1}{4}$

The line is $y = \frac{5}{3}x$



The inequality is $y \geq \frac{5}{3}x$