Definitions

A set of ordered pairs, $\{(x, y)\}$ is called a *function* if the set has the property that no two ordered pairs have the same x-coordinate.

The set of all of the x-coordinates is called the *domain* of the function.

The set of all of the y-coordinates is called the *range* of the function.

If the name of the function is f, and one of the ordered pairs in the set is (a, b), we say that $f(a) = b$. "f at a equals b" "f of a equals b"

The value of f at a is b; b is called the function value.

Determine whether each of the following sets is a function. If the set is a function, state the domain of the function, the range of the function, and the values of $f(-3)$ and $f(4)$ (assuming that the function name is f). If the set isn't a function, state at least one example that indicates that it is not a function.

1. The set of points $\{(9,0),(-3,7),(4,11),(-14,-7)\}$.

This set of points is a function

Domain: $\{9, -3, 4, -14\}$

Range: $\{0, 7, 11, -7\}$

$f(9) = 0$ $\quad$ $f(4) = 11$

$f(-3) = 7$ $\quad$ $f(-14) = -7$

2. The set of points $\{(8,9),(-3,9),(5,8),(3,-7)\}$.

This is a function

Domain: $\{8, -3, 5, 3\}$

Range: $\{9, 8, -7\}$

$f(-3) = 9$

$f(4)$ is not defined.

$f(4)$ does not exist

4 is not in the domain of the function

3. The set of points $\{(9,0),(-2,7),(3,18),(9,-22)\}$.

NOT A FUNCTION

Domain	Range
9	0
-2	7
3	18
	-22

$9 \to 0$, $9 \to -22$, $-2 \to 7$, $3 \to 18$

9 is mapped to two different numbers.

Is $f(9) = 0$ or is $f(9) = -22$??

4. The set of points $\{(2.3,89),(-3,89),(4.01,89),(9,89)\}$

This is a constant function.

$f(\text{any number in the domain}) = 89$

Domain: $\{2.3, -3, 4.01, 9\}$

Range: $\{89\}$

4. The set of points on the curve shown in Figure 1.

This is not a function.

For example, there are — count 'em — four points where $x = -2$.

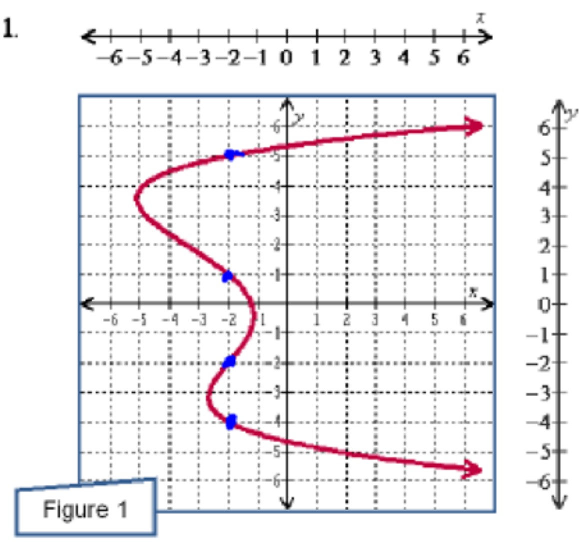
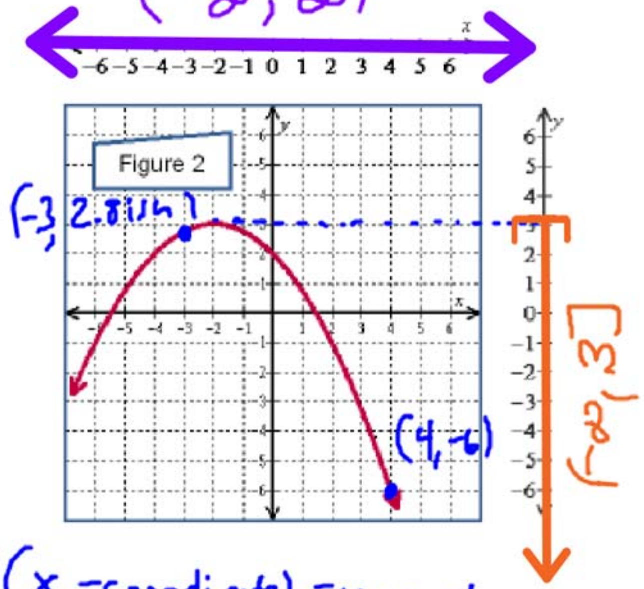


Figure 1

5. The set of points on the curve shown in Figure 2.

$(-\infty, \infty)$



f(x-coordinate) = y-coordinate

This is a function.

The range is $(-\infty, 3]$.

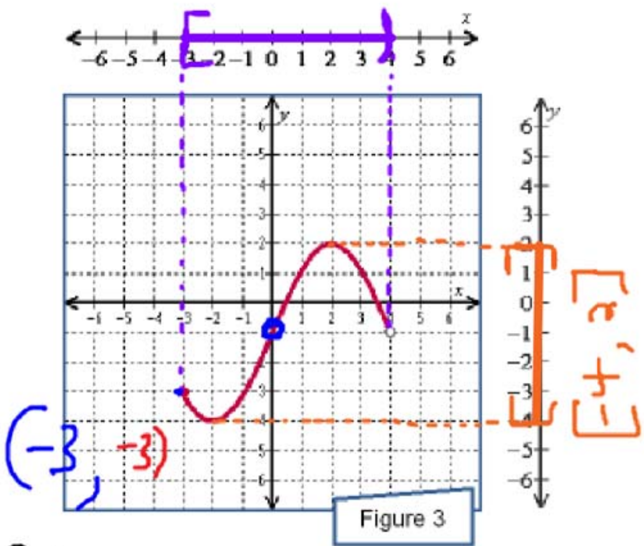
The domain is $(-\infty, \infty)$.

i.e. The domain is $\mathbb{R}$.

$f(4) = -6$, $f(-3) \approx 2.8$

$\{x \mid -3 \leq x < 4\}$

6. The set of points on the curve shown in Figure 3.



$f(-3) = -3$

This is a function.

The domain is $[-3, 4)$.

The range is $[-4, 2]$.

$f(4)$ is undefined.

The open circle means the point $(4, -1)$ is not on the function. If that circle was filled in, then $f(4) = -1$.

f

7. The set of points $\{(x, y) \mid y = 2\}$

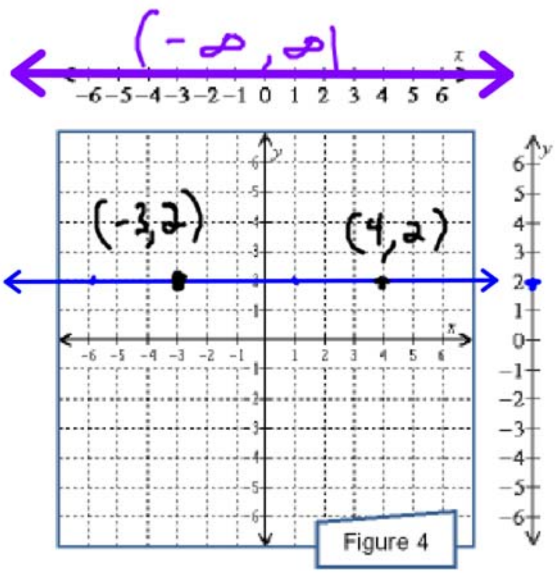


Figure 4

This is a function

The domain is $(-\infty, \infty)$

The range is $\{2\}$

$f(-3) = 2$

$f(4) = 2$

In fact,

$f(\text{any } \#) = 2$.

we write $f(x) = 2$.

no matter what the value of x is.

Function notation and formulas

One thing we need to learn about functions is the way in which the notation works. We already talked about the way it works when you are given a set or a graph. Now we're going to look at the way it works when given a formula. Suppose that you wanted to evaluate the expression $3x^2$ for various values of x. One way you could notate your work is as follows.

When $x = 4$, $3x^2 = 3 \cdot 4^2$
$= 3 \cdot 16$
$= 48$

When $x = -2$, $3x^2 = 3 \cdot (-2)^2$
$= 3 \cdot 4$
$= 12$

Function notation gives us a way to present the work without words. We give the function a name, most commonly f, and state the rule for f using another letter, most commonly x. In the formula above, we would write $f(x) = 3x^2$. Note that this gives us a place to the left of the equal sign to explicitly state the value of x without having to write "When $x =$ whatever." From the work above, we see that for this function

$$f(4) = 48 \text{ and } f(-2) = 12$$

In this example, 4 and -2 are called inputs to the function and their respective outputs are 48 and 12.

Let's go ahead and find $f(7)$ and $f(-1)$ for the function $f(x) = 3x^2$

$f(x) = 3x^2$

$f(7) = 3(7)^2$

$= 3(49)$

$= 147$

$f(-1) = 3(-1)^2$

$= 3(1)$

$= 3$

Find $g(-6)$ and $g(0)$ if $g(x) = 3 - x^2$.

The function name is g.

$g(x) = 3 - x^2$

$g(-6) = 3 - (-6)^2$

$= 3 - 36$

$= -33$

$g(0) = 3 - 0^2$

$= 3$

The function value at 0 is 3.

The point (0, 3) is on the graph of this function!

Find $f(10)$ and $f(1)$ if $f(t) = \frac{-t^2}{2}$.

The function name is f.

$f(t) = \frac{-t^2}{2}$

$f(10) = \frac{-(10)^2}{2}$

$= \frac{-100}{2}$

$= -50$

$f(1) = \frac{-(1)^2}{2}$

$= -\frac{1}{2}$

Find $g(11)$ and $g(-2)$ if $g(t) = (t+2)^2$

$g(t) = (t+2)^2$

$g(11) = (11+2)^2$

$= 13^2$

$= 169$

$g(-2) = (-2+2)^2$

$= 0^2$

$= 0$

Find $f(11)$ and $f(-2)$ if $f(t) = 3t - 9$

$f(t) = 3t - 9$

$f(11) = 3(11) - 9$

$= 33 - 9$

$= 24$

$f(-2) = 3(-2) - 9$

$= -6 - 9$

$= -15$

Find $h(0)$ and $h(-4)$ if $h(x) = 29$

$h(x) = 29$

$h(0) = 29$

$h(-4) = 29$