

$$x^2 x^5$$

$$x^2 x^5 = \underbrace{(xx)(xxxxx)}_{2+5 \text{ factors of } x}$$

$$x^2 x^5 = x^{2+5} = x^7$$

$$a^3 a$$

$$a^3 a = \underbrace{(aaa)(a)}_{3+1 \text{ factors of } a}$$

$$a^3 a = a^3 a^1 = a^{3+1} = a^4$$

$$y^3 y^3$$

$$y^3 y^3 = \underbrace{(yyy)(yyy)}_{3+3 \text{ factors of } y}$$

$$y^3 y^3 = y^{3+3} = y^6$$

$$w^{27} w^{42}$$

$$w^{27} w^{42} = w^{27+42} = w^{69}$$

The product rule for exponents:  $x^m x^n = x^{m+n}$

Use properties of exponents to help you simplify each of the following

$x^{44} x^{17}$

$$\begin{aligned} x^{44} x^{17} &= x^{44+17} \\ &= x^{61} \end{aligned}$$

$(a^4 b^2)(a^7 b)$

$$\begin{aligned} (a^4 b^2)(a^7 b) &= a^{4+7} b^{2+1} \\ &= a^{11} b^3 \end{aligned}$$

$(3x^6)(-2x^5)$

$$\begin{aligned} (3x^6)(-2x^5) &= (3 \cdot -2)(x^6 x^5) \\ &= -6x^{6+5} \\ &= -6x^{11} \end{aligned}$$

$(4y^{12})\left(\frac{y^{12}}{4}\right)$

$$\begin{aligned} (4y^{12})\left(\frac{y^{12}}{4}\right) &= \frac{4}{4} y^{12} y^{12} \\ &= 1 \cdot y^{12+12} \\ &= y^{24} \end{aligned}$$

Completely simplify each of the following.

$$\begin{aligned} (x^2)^3 &= (x^2)(x^2)(x^2) \\ &= \underbrace{(xx)(xx)(xx)}_{3 \text{ times } 2 \text{ factors of } x} \end{aligned}$$

$$\begin{aligned} (t^4)^2 &= (t^4)(t^4) \\ &= \underbrace{(tttt)(tttt)}_{2 \text{ times } 4 \text{ factors of } t} \end{aligned}$$

$$\begin{aligned} (w^3)^2 &= w^{3 \cdot 2} \\ &= w^6 \end{aligned}$$

$$\begin{aligned} (u^{20})^4 &= u^{20 \cdot 4} \\ &= u^{80} \end{aligned}$$

The power rule for exponents:  $(x^m)^n = x^{mn}$

# Exponents distribute over multiplication and division

Mr. Simonds' MTH 60 class

$\begin{matrix} P \\ E \\ MD \\ AS \end{matrix}$  distribute  $\rightarrow$  distribute

Completely simplify each of the following.

$$\begin{aligned}(ab)^4 &= (ab)(ab)(ab)(ab) \\ &= (aaaa)(bbbb) \\ &= a^4b^4\end{aligned}$$

$$\begin{aligned}(2x)^3 &= (2x)(2x)(2x) \\ &= (2 \cdot 2 \cdot 2)(xxx) \\ &= 2^3x^3 \\ &= 8x^3\end{aligned}$$

Product to a power rule for exponents:  $(xy)^n = x^n y^n$

Completely simplify each of the following.

$$\begin{aligned}(4x^5)^3 &= 4^3(x^5)^3 \\ &= 4^3x^{5 \cdot 3} \\ &= 64x^{15}\end{aligned}$$

$$\begin{aligned}
 (-2w^2)^4 &= (-2)^4 (w^2)^4 \\
 &= 16 w^{2 \cdot 4} \\
 &= 16 w^8
 \end{aligned}$$

$$\begin{aligned}
 (3t^{12})(-t^{64}) &= -3 t^{12+64} \\
 &= -3 t^{76}
 \end{aligned}$$

$$\begin{aligned}
 \left(\frac{1}{2}r^7\right)^5 &= \left(\frac{1}{2}\right)^5 (r^7)^5 \\
 &= \left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right) r^{7 \cdot 5} \\
 &= \frac{1}{32} r^{35}
 \end{aligned}$$

$$\begin{aligned}
 (-2s^4t)^7 &= (-2)^7 (s^4)^7 t^7 \\
 &= -128 s^{4 \cdot 7} t^7 \\
 &= -128 s^{28} t^7
 \end{aligned}$$

### Proportions

An equation that equates two fractions,  $\frac{a}{b} = \frac{c}{d}$ , is called a proportion; in this context each fraction is called a ratio. *Proportions and ratios are one of the most commonly used mathematical concepts in modern day society!*

Solve each proportion

$$\frac{x}{2} = \frac{7}{5}$$

$$\frac{x}{2} = \frac{7}{5}$$

SCRATCH WORK  $\rightarrow 5 \cancel{10} \left( \frac{x}{2} \right) = \cancel{10} \left( \frac{7}{5} \right)$

$$5x = 2(7)$$

$$5x = 14$$

$$x = \frac{14}{5}$$

$$\frac{4}{t} = \frac{9}{2.5}$$

$$\frac{4}{t} = \frac{9}{2.5}$$

$$4(2.5) = 9t$$

$$10 = 9t$$

$$\cancel{9}9 = t$$

$$\frac{y - y_1}{x - x_1} = \frac{m}{1} \quad \text{solve for } y$$

$$\frac{y - y_1}{x - x_1} = \frac{m}{1}$$

$$(1)(y - y_1) = (m)(x - x_1)$$

$$y - y_1 = m(x - x_1)$$

$$y = m(x - x_1) + y_1$$

$$y = mx - mx_1 + y_1$$

$$y = mx + \underbrace{(-mx_1 + y_1)}_b$$

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Cross  
multiplication

Four out of every 73 rock albums sold in 1977 was recorded by the Eagles. The Eagles sold a total of 2.6 million albums in 1977. How many rock albums total were sold in 1977.

Define your variable to be the **total number of rock albums sold.**

The equation comes from the proportion.

Solve and check your equation and then state your conclusion using a complete sentence.

Let  $x$  be the total # of rock albums sold in 1977.

Choose  
our ratio

$\frac{\text{Eagles}}{\text{total}}$  OR  $\frac{\text{total}}{\text{Eagles}}$   
★

$$\frac{4}{73} = \frac{2,600,000}{x}$$

$$4x = 73(2,600,000)$$

$$x = \frac{73(2,600,000)}{4}$$

$$x = 47,450,000$$

A total of 47,450,000  
rock albums were sold  
in 1977





Even goldfishes need their space; specifically, fish people recommend that you have no more than 3 goldfish per gallon of water in your tank. Suppose that you buy a spherical tank with a radius of 10 inches. What is the maximum number of goldfish you should put into this tank?

Use the formula  $V = \frac{4}{3}\pi r^3$  to find the volume of the tank and then convert the volume to gallons.

There are 231 in<sup>3</sup> in one gallon.

$$\text{ratio: } \frac{\# \text{ of fish}}{\text{volume}} \quad (3 \text{ goldfish per gallon})$$

We need to know the volume of the spherical tank (in gallons)

$$V = \frac{4}{3}\pi (10 \text{ inches})^3$$

$$\approx 4188.8 \text{ in}^3 \quad (V \text{ is volume of tank})$$

$$\frac{1 \text{ gal}}{231 \text{ in}^3} = \frac{V}{4188.8 \text{ in}^3} \quad \left| \quad V = \frac{4188.8}{231} \text{ gal} \right.$$

$$\approx 18.1 \text{ gal}$$

Let  $x$  be the maximum # of fishes in the tank

$$\frac{3 \text{ fishes}}{1 \text{ gal}} = \frac{x}{18.1 \text{ gal}}$$

$$(1 \text{ gal})x = (3 \text{ fishes})(18.1 \text{ gal})$$

$$x = 54.3 \text{ fishes.}$$

We can have at most 54 fishes.

Dude, major space violation!

