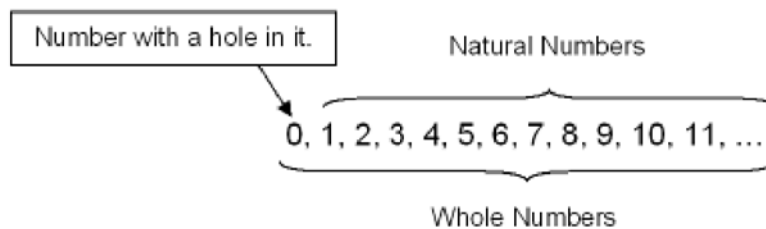


The Real Number Line and Types of Real Numbers

Most people's first understanding of numbers relates to what we math-type people call counting numbers or - more formally - natural numbers. These are the numbers we use to count 1 or more whole occurrences of similar objects. We also use these numbers to keep our young-uns busy when they've sung one too many renditions of the alphabet song.

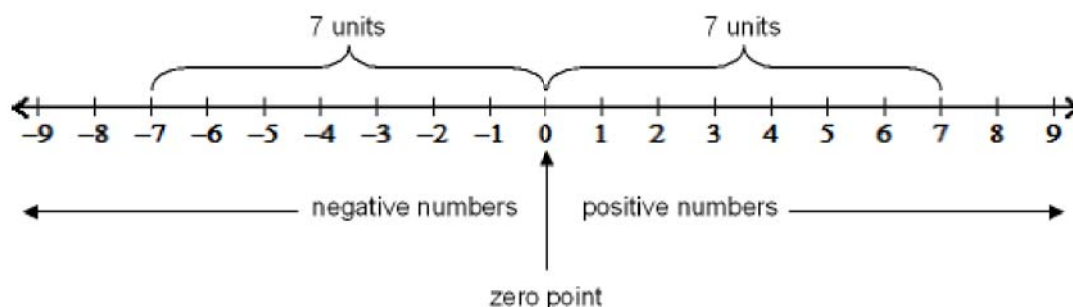
Pretty soon after this, we generally form an understanding of the number zero. When we add the number zero to the set of natural numbers we create a set of numbers we call the whole numbers.



The next type of numbers we generally come to understand are fractions, but we'll put these on hold for now.

One day in your past, someone rocked your world with the idea of negative numbers. At first these seem very strange - how can you have a negative number of chickens? The answer, of course, is that you can't. So negative numbers don't make sense when counting chickens... that's why they're not part of the set we call counting numbers.

You can, however, be above or below sea level. If we call sea level an elevation of zero, one easy way to distinguish points above sea level from points below sea level is to assign positive values to points above sea level and negative values to points below sea level. If we're measuring in feet, for example, 7 would correspond to a point that is 7 feet above sea level while -7 would correspond to a point that is 7 feet below sea level. The numbers 7 and -7 are called opposite numbers because they are equidistant from zero and have opposite signs. One easy way to illustrate this is with a number line.



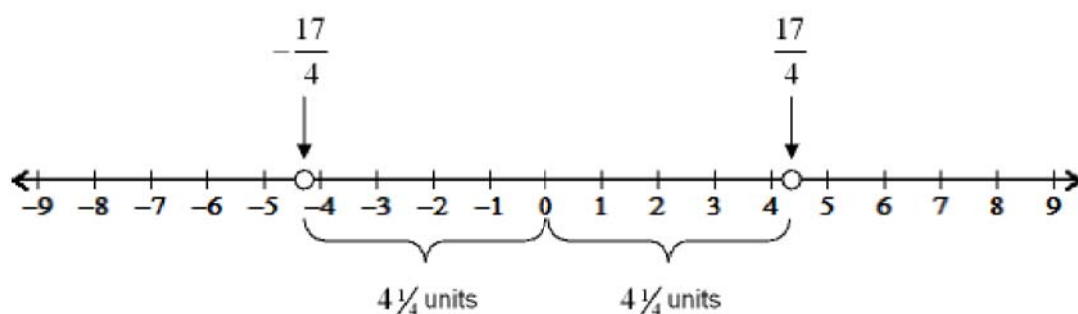
Example 1

Complete each sentence *correctly*. ☺

The opposite of -12 is 12. The opposite of 0 is 0.

The set of numbers consisting of the natural numbers, 0, and the opposites of the natural numbers is called the set of integers.

OK... back to fractions. Generally, when we think of fractions we think of one positive integer divided by another positive integer. We represent half of a pie with the fraction $\frac{1}{2}$. We represent 2 out of 3 voters with the fraction $\frac{2}{3}$. Well, just as natural numbers have opposites, so do positive fractions. For example, $\frac{17}{4}$ and $-\frac{17}{4}$ are opposite numbers.



Any number that *can* be written as a quotient of *integers* is called a rational number. Remember that a fraction cannot have a denominator of zero!

$$\text{rational \#} \rightarrow \frac{\text{integer}}{\text{integer}}$$

Example 3

Show that the numbers 0, 8.3, $\sqrt{25}$, and $0.333\bar{3}$ are all rational numbers.

$$0 = \frac{0}{3}$$

$$8.3 = \frac{83}{10}$$

$$\begin{aligned} \sqrt{25} &= 5 \\ &= \frac{25}{5} \text{ or } \frac{10}{2} \text{ or } \frac{5}{1} \end{aligned}$$

$$\begin{aligned} 0.333\bar{3} &= \frac{1}{3} \text{ or } \frac{11}{33} \\ &\text{or } \frac{8}{24} \end{aligned}$$

Every point on the real number line corresponds to a unique real number and every real number corresponds to a unique point on the real number line. Every real number is either a rational number or an irrational number.

Example 4

Place each of the following numbers into the sets (boxes) to which they belong. Note that most of the numbers fit into more than one set.

 $\sqrt{-4}$

$$\cancel{\frac{15}{2}}, -16.812, \frac{28}{7}, -14, \sqrt{5}, \sqrt{25}, \sqrt{-4}, \cancel{0}, \frac{\pi}{3}, 9.22124457$$

Real numbers

$$\frac{15}{2}, 0, -16.812, \frac{28}{7}, -14, \sqrt{5}, \sqrt{25}, \frac{\pi}{3}, 9.22124457$$

Rational numbers

$$\frac{15}{2}, \sqrt{25}, 0, 9.22124457, -16.812, \frac{28}{7}, -14$$

Irrational numbers

$$\sqrt{5}, \frac{\pi}{3}$$

Integers

$$0, \frac{28}{7}, -14, \sqrt{25}$$

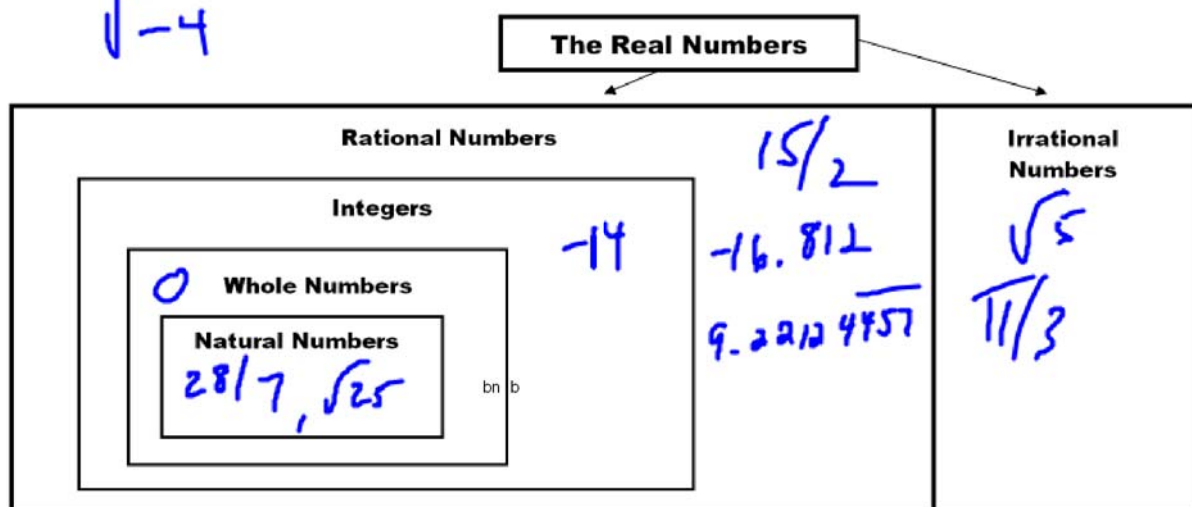
Whole numbers

$$0, \frac{28}{7}, \sqrt{25}$$

Natural numbers (Counting numbers)

$$\frac{28}{7}, \sqrt{25}$$

$$\sqrt{-4}$$



The real numbers: The set of numbers that correspond to points on the real number line.

Rational number: Any number that can be written as a fraction of integers. In decimal form, a rational number either terminates or forms a forever repeating pattern.

Irrational number: Any real number *that is not a rational number*. π is an irrational number. The square root of a natural number is either itself a natural number (like $\sqrt{49}$) or is an irrational number (like $\sqrt{15}$).

Integer: Any number that simplifies to a number in the list: ... -4, -3, -2, -1, 0, 1, 2, 3, 4, ...

Whole number: Any number that simplifies to a number in the list: 0, 1, 2, 3, 4, ...

Natural number: Any number that simplifies to a number in the list: 1, 2, 3, 4, ...

Example 5

Decide whether each statement is true or false. If the statement is false, give an example that shows why the statement is false.

- Every integer is a natural number.

False

- Every whole number is a rational number.

True

- Some natural numbers are irrational numbers.

False

Equality and Inequalities

As previous mentioned, every real number corresponds to a unique point on the real number line. When plotting two *different* numbers on a traditional number line, one of the two numbers lies to the left of the other number. The number on the left is called the lesser of the two numbers and the number on the right is called the greater of the two number.

$a < b$ means that a lies to the left of b along the real number line. The expression " $a < b$ " is read aloud as " a is less than b ."

$a > b$ means that a lies to the right of b along the real number line. The expression " $a > b$ " is read aloud as " a is greater than b ."

$a = b$ means that a and b are in fact the same point along the real number line. The expression " $a = b$ " is read aloud as " a equals b ."

$a \leq b$ means that $a < b$ or $a = b$. The expression " $a \leq b$ " is read aloud as " a is less than or equal to b ."

$a \geq b$ means that $a > b$ or $a = b$. The expression " $a \geq b$ " is read aloud as " a is greater than or equal to b ."

$a < b$, $a > b$, $a \leq b$, and $a \geq b$ are called inequalities (or inequality statements).

$a = b$ is called an equation.

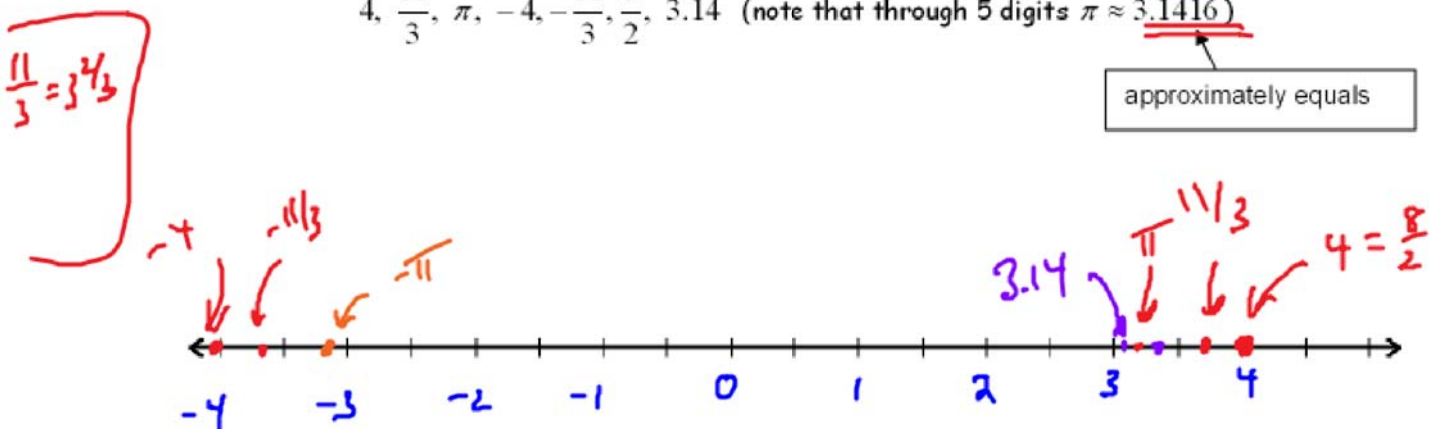
Example 6

Indicate, roughly, the position of each of the following numbers on the provided number line *after first deciding on an appropriate scale* and then answer the subsequent questions using proper mathematical symbols and notation.

$$\pi \approx 3.1400 + .0016$$

$$4, \frac{11}{3}, \pi, -4, -\frac{11}{3}, \frac{8}{2}, 3.14 \quad (\text{note that through 5 digits } \pi \approx \underline{\underline{3.1416}})$$

approximately equals



Scale: $\frac{1}{2}$ (distance between tick marks)

Which is the true statement: $\frac{11}{3}$ is less than 4 or 4 is less than $\frac{11}{3}$?

$$\frac{11}{3} < 4$$

Which is the true statement: $-\frac{11}{3}$ is less than -4 or -4 is less than $-\frac{11}{3}$?

$$-4 < -\frac{11}{3}$$

$$-\frac{11}{3} > -4$$

Which is the true statement: $\pi = 3.14$, $\pi > 3.14$, or $\pi < 3.14$?

$$\pi > 3.14$$

Which is the true statement: $-\pi = 3.14$, $-\pi > 3.14$, or $-\pi < 3.14$?

$$-\pi < 3.14$$

Which is true: $-\frac{11}{3} = -\pi$, $-\frac{11}{3} \leq -\pi$, or $-\frac{11}{3} \geq -\pi$?

$$-\frac{11}{3} \leq -\pi$$

Which is true: $\frac{8}{2} = 4$, $\frac{8}{2} \leq 4$, or $\frac{8}{2} \geq 4$?

All three are true!!

Absolute Value

The symbols $|n|$ are read aloud as "the absolute value of n ." The absolute value of the real number n is the distance between that number and the number zero along the real number line.

Fact 1: The *absolute value* of a number is *never negative*.

Fact 2: Opposite numbers have equal absolute values.

Example 8

For each given pair, decide which is the true statement.

$-|-9| = -9$ $|-9| = 9$ or $|-9| = -9$
 $-(-9) = 9$ $|9| = 9$ or $|9| = -9$ The opposite of a positive (+9) is a negative number
 $-|-9| = 9$ or $-|-9| = -9$
 $-|9| = 9$ or $-|9| = -9$

Example 9

Without using your calculator, determine which of the following is true.

$$\sqrt{5} < 2.3 \quad \text{or} \quad \sqrt{5} = 2.3 \quad \text{or} \quad \sqrt{5} > 2.3$$

$$\begin{array}{r}
 2.3 \\
 2.3 \\
 \hline
 469 \\
 460 \\
 \hline
 5.29
 \end{array}$$

$$\sqrt{5} < 2.3$$