Q: Just what the heck is algebra?

A: Truth be told, algebra is a whole lot of things. In fact, I'm certain that several students in this class could come up with all sorts of words to describe algebra.

To me (me being Mr. Simonds), at it heart algebra is a language system based upon letters representing numbers. The letters we use to represent numbers are called ***variables***. Sometimes a variable can represent every real number all at once. Sometimes a variable represents some numbers but not others. Sometimes a variable represents one specific, yet unknown, number.

Using variables to represent numbers afford us (us being humankind) the opportunity to achieve all sorts of things. For example:

Variables allow us to make broad statements that apply to all sorts of numbers.

An example would be ***the commutative property of multiplication***.: if a and b represent any real numbers, then $a \times b = b \times a$.

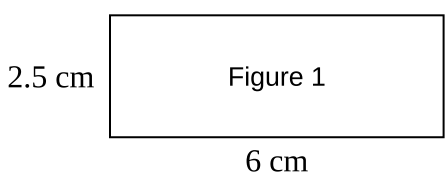
For example, $2 \times 3 = 3 \times 2$. Now you might think this is "obvious" because 2×3 and 3×2 mean the same thing. But this just isn't so.

Example 1

What is the difference between 2×3 and 3×2?

Variables allow us to efficiently communicate calculation formulas.

For example, if l and w represent the length and width, respectively, of a rectangle, then the area of the rectangle, A, can be determined using the formula $A = l \times w$.



Example 2

Use the formula $A = l \times w$ to determine the area of the rectangle shown in Figure 1.

Letting $l = 6$ cm and $w = 2.5$ cm we have:

Variables allow us to efficiently state questions about unknown numbers.

Many of these questions are stated using ***equations*** and the more math classes you take the more rules you learn that enable you to ***solve the equation*** and, consequently, determine the unknown numbers.

Example 3

Write an equation that represents the following question: To what number do you add 7 and end up with 34?

We're not going to get anywhere unless we formally define our ***variable***. In this simplified example, we can define x to be the unknown number.

In order to have clear communication about algebra, we need to have a common vocabulary related to algebra and we need to have a common set of rules for how we write algebraic expressions. We also need to agree upon the format we use when writing algebraic work; this last agreement is akin to the way in which we agree to organize sentences into paragraphs and paragraphs into papers.

Lucky for you (the student) all of these definitions, syntax, and organizational strategies have been settled long before you came onto the scene; all you need to do is learn the rules that have already been established.

Some of you are going to have a gut instinct that tells you to fight against the rules; this is most likely to happen when we get to the "paragraph" level … that is, when I tell you how to organize your work. While I can understand the instinct to rebel, you should know that resistance is futile. Part of my job is to teach you the proper way these things are done and I will expect you to write your work using correct notation and proper organizational strategies.

Example 4

What does the expression $\frac{3}{2}x$ mean in an algebraic context?

Example 5

What is the value of $\frac{3}{2}x$ when x has the value of 16?

Notice how the symbols align. For example, where does the *x* fall relative to the fraction? Is it next to the 3? Is it next to the 2? What is it next to?

Let's use mathematical symbols, when available, to efficiently communicate our thoughts.

When you write your work ***write it using the same organizational strategy that is being shown by Mr. Simonds***. Part of what you are going to ***learn*** in this class is appropriate strategies for organizing mathematical work. To keep things simple, we are all going to use the same organization … as in "yes you will be graded on this." You need to learn correct syntax for mathematics (this is truly a "right" or "wrong" issue) and correct ways to organize your work. There is not "universal" agreement about the best way to organize work – that is, one teacher will say one thing and another something else. In this class you need to write it the way Mr. Simonds writes it. When you see slight differences in the way the book shows the work and the way Mr. Simonds shows it, go with Mr. Simonds. I'm not talking about "how to do the problem" here …. I'm talking about how to organize your work.

What's written below is something that would be considered scratch work …. It's OK to write stuff like this – heck, we all do scratch work - but box it off from where you're showing your "formal" problem solution.

Example 7

What is the value of $2+4t$ when t has a value of 3?

We perform the multiplication before the addition because of ***order of operations***.

Example 8

- Introduce yourself to at least one other student in the class (someone sitting close to you!).
- Together with your new found classmate(s), work each of the following problems ***showing work consistent with that shown in examples 6 and 7***. Your goal is not simply to "figure out the number." Your goal is also to start learning the way you need to write you work in an algebraic environment (generally) and Mr. Simonds class (specifically).

What is the value of $75-4x$ when x has a value of 12?

What is the value of $\frac{4}{7}y$ when y has a value of 35?

The calculations we performed in examples 6, 7, and 8 have a formalized description in algebra. The process is called ***evaluating an expression***. Unfortunately for you, many of the fundamental words in algebra start with the letter "e," so you're going to have to make a conscious effort to memorize what the words mean.

Example 9

Evaluate the expression $xy - 6y$ when $x = 20$ and $y = \frac{1}{2}$.

Can you see a clue in one of the words that tells you that you are suppose to find the value of something?

Definition

An algebraic expression is a meaningful arrangement of mathemematical symbols including things such as numbers, variables, multiplication, division, addition, subtraction, exponents, and radicals but excluding equal signs or inequality signs. ***If the symbols include an equal sign or inequality sign the arrangement is not called an expression.***

Example 10

Decide which of the following are algebraic expressions. State the reason those that aren't algebraic expressions are in fact not algebraic expression

a. $5x^2 - \frac{4}{z}$

b. $3x + 2 = 7$

c. $\frac{5}{x+} - \sqrt{}$

d. 

Language Lesson …. An important language lesson!

You cannot *solve* an expression – there is nothing to solve.

You can *evaluate* an expression or **you can *simplify* an expression**.

You can *manipulate* an expression. You can *change* an expression. You can *replace* an expression with an equivalent expression. You can *add two* to an expression. You can *multiply an expression by 17*. There are all sorts of things you can do to an expression. There is one thing you can never do, however.

You cannot *solve* an expression – there is nothing to solve.

Example 11

Colin went to the tutor center and asked a tutor to help him solve $70-4x$. The tutor didn't know what Colin meant. Why is that?

Example 12

Once the tutor figured out what Colin really needed to do he helped Colin come up with the following. Talk with you classmate about the proper language that describes what Colin did and write a sentence that reflects the process.

Colin's work

When $x=7$:

$$\begin{aligned} 70-4x &= 70-4(7) \\ &= 70-28 \\ &= 42 \end{aligned}$$

Well, by golly, there's got to be something you ***can solve***. You'd be right about that … one thing you can solve is an equation.

Definitions

Two algebraic expressions separated by an equal sign is called, in aggregate, an ***equation***.

Sometimes an equation contains a single variable that represents an unknown number. Determining the number you replace the variable with that makes the expressions on both sides of the equal sign evaluate to a common value is called ***solving the equation*** and the number that does the trick is called ***the solution to the equation***.

Example 13

What is the solution to the equation $t+9=15$?

Example 14

True or false: 8 is a solution to the equation $\frac{2+3x}{60-x}=\frac{x+4}{3x}$?

Example 15

Mr. Simonds was asked the following question: "Solve $7x$ when $x=9$." Mr. Simonds had issues with the question. What was Mr. Simonds' beef?

Example 16

Write an expression or equation that represents each of the following statements or sentences. In each case, state whether you use an expression or an equation. You can use x to represent each unknown number.

6 less than a number

The product of 2 and a number is equal to 7.

Half a number is the same as 3 more than the number.

the quotient of a number and 5

7 less than 3 times a number is equal to the number itself!

Example 17

Tito was taking a test and was asked to determine the solution to the equation $x+1=9$. Tito wrote "The solution is $x=8$." While grading the papers, Mr. Simonds wrote, "You have the right idea, Tito, but your answer isn't quite right." What was stuck in Mr. Simonds' craw?

Example 18

On the same test Emma was asked to evaluate $5t+1$ when $t=\frac{1}{5}$ and the last thing Emma wrote was "The solution is 2." Mr. Simonds wrote, "You did a good job of calculating, Emma, but you're not writing things correctly." What's the bee in Mr. Simonds' bonnet this time?

Example 19

If your taxable income ($) for a year, d, is over $7500, then your Oregon state income tax for that year ($) is given by the formula $I=471+.09(d-7500)$. Determine how much Income tax you owe the state of Oregon in a year your taxable income is $10,000.

Example 20

Evaluate $2x+\frac{3}{2}$ when $x=16\tfrac{2}{3}$. Write your final value as a mixed number.

Example 21

Evaluate $3t-\frac{7}{4}$ when $t=17\tfrac{1}{2}$. Write your final value as an improper fraction.

Example 22

Evaluate $\frac{5}{z}$ when $z=6\tfrac{7}{9}$. Write your final value as an improper fraction.

Example 23

Evaluate $\frac{4/7}{y}$ when $y=2\tfrac{9}{14}$.