1. An eigenvector for each of the following matrices is given. In each case, determine the eigenvalue associated with the given eigenvector. **No calculator usage allowed.**

   a. $\begin{bmatrix} 18 \\ 6 \\ 1 \end{bmatrix}$ is an eigenvector for $\begin{bmatrix} 0 & 4 & 3 \\ 1/2 & 0 & 0 \\ 0 & 1/4 & 0 \end{bmatrix}$

   b. $\begin{bmatrix} -5 \\ 1 \\ 7 \end{bmatrix}$ is an eigenvector for $\begin{bmatrix} 1 & -2 & 1 \\ -3 & -1 & -2 \\ -7 & 7 & -6 \end{bmatrix}$

2. Use the characteristic equation $\det\left(A-\lambda I\right)=0$ to determine the eigenvalues for each of the given matrices. **No calculator usage allowed.**

   a. $A=\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

   b. $A=\begin{bmatrix} 0 & 4 \\ -1 & 5 \end{bmatrix}$

3. The eigenvalues for each of the following matrices are given. In each case, use the equation $\left(A-\lambda I\right)\vec{x}=\vec{0}$ to help you determine bases for the associated eigenspaces. You may use your calculator to row reduce a matrix when row reduction is necessary.

   a. The eigenvalues for $A=\begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$ are 3 and -2 .

   b. The eigenvalues for $A=\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$ are 3, 1 and -1.

4. The eigenvalues and bases for the associated eigenspaces for the matrix $B=\begin{bmatrix} 7 & 1 & -2 \\ -3 & 3 & 6 \\ 2 & 2 & 2 \end{bmatrix}$ are 0 with basis $\left\{\begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}\right\}$ and 6 with basis $\left\{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix},\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right\}$. Use this information to determine a diagonalization of B; i.e., determine matrices P and D such that $B=PDP^{-1}$ where D is a diagonal matrix. Verify your result by hand. You may use your calculator to determine P^{-1}.

5. Let $A=\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$. Find a formula for A^k. Then use your formula to determine A^{10} and verify the result with your calculator. Note : to begin you need to diagonalize A. **No calculator usage allowed.**