nul $(A) \subset \mathbb{R}^6$

Col $(A) \subset \mathbb{R}^4$

Let $A = \begin{bmatrix} 2 & 1 & -4 & 3 & -2 & -5 \\ -1 & 1 & 2 & 3 & 1 & -11 \\ 1 & 4 & -2 & -3 & -1 & 11 \\ -2 & -2 & 4 & -1 & 2 & -1 \end{bmatrix}$. Then $A \sim \begin{array}{c} \begin{array}{cccccc} p & p & & p & & \end{array} \\ \begin{bmatrix} 1 & 0 & -2 & 0 & -1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{cccccc} & & \text{free} & & \text{free} & \text{free} \end{array} \end{array}$. $\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}$

What are the dimensions of the null space and column space of A and how do you know. What do these dimensions sum to?

The dimension of a subspace of $\mathbb{R}^n$ is not necessarily n.

The dimension is the number of vectors in a basis of the subspace.

A has three pivot columns, so $\dim(\text{col}(A)) = 3$.

a basis for col (A) is $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -3 \\ -1 \end{bmatrix} \right\}$

The solution set to $A\vec{x} = \vec{0}$ has three free variables (x_3, x_5, and x_6) so $\dim(\text{nul}(A))$ is three

$3 + 3 = 6$ $\quad \dim(\text{col}(A)) + \dim(\text{nul}(A)) = \#$ of columns of A

State bases for the null space and column space of A.

oops, already did that

The general solution to $A\vec{x} = \vec{0}$ is $\left\{ \begin{array}{l} x_1 = 2x_3 + x_5 - 2x_6 \\ x_2 = 0 \\ x_3 \text{ is free} \\ x_4 = 3x_6 \\ x_5 \text{ is free} \\ x_6 \text{ is free} \end{array} \right.$

Solutions can be written as $x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -2 \\ 0 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$

$\therefore$ A basis for nul (A) is:

$$\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

MTH 261 – Mr. Simonds' class

Determine the dimension of span$\left(\left\{\begin{bmatrix}1\\-1\\0\\4\\-2\end{bmatrix},\begin{bmatrix}-4\\4\\0\\-16\\8\end{bmatrix},\begin{bmatrix}-1\\2\\1\\3\\-2\end{bmatrix},\begin{bmatrix}1\\0\\1\\11\\-6\end{bmatrix},\begin{bmatrix}-2\\3\\1\\-1\\0\end{bmatrix}\right\}\right)$. Explain your thought process!

The dimension is the number of vectors in a basis which determine by analyzing the column space of A

where $A = \begin{bmatrix} 1 & -4 & -1 & 1 & -2 \\ -1 & 4 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 & 1 \\ 4 & -16 & 3 & 11 & -1 \\ -2 & 8 & -2 & -6 & 0 \end{bmatrix}$

$A \sim \begin{bmatrix} 1 & -4 & 0 & 2 & -1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ A has two pivot columns, so $\dim(\mathrm{col}(A)) = 2$

The span of the set is two-dimensional.

Any two vectors from that set form a basis for the span of the set, except $\{\vec{c}_1, \vec{c}_2\}$ (C_1 and C_2 are multiples hence linearly dependent)

Find a basis for the null space of B where $B = \begin{bmatrix} -i & 1 \\ -1 & -i \end{bmatrix}$.

Solve $B\hat{x} = \vec{0}$.

$\left[\begin{array}{cc|c} -i & 1 & 0 \\ -1 & -i & 0 \end{array}\right]$ $iR_1 + R_2 \to R_2$ $\left[\begin{array}{cc|c} -i & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

Clearly $x_2 = i x_1$. Letting $x_1 = i$,

a basis for $\mathrm{nul}(B)$ is $\left\{\begin{bmatrix} i \\ -1 \end{bmatrix}\right\}$.