Let $A = \begin{bmatrix} 2 & 1 & -4 & 3 & -2 & -5 \\ -1 & 1 & 2 & 3 & 1 & -11 \\ 1 & 4 & -2 & -3 & -1 & 11 \\ -2 & -2 & 4 & -1 & 2 & -1 \end{bmatrix}$. Then $A \sim \begin{array}{c} \begin{array}{cccccc} P & P & F & P & F & F \end{array} \\ \begin{bmatrix} 1 & 0 & -2 & 0 & -1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$.

What are the dimensions of the null space and column space of A and how do you know. What do these dimensions sum to?

$\dim(\text{null}) = \#$ of free variables
$= 3$

$\dim(\text{col}) = \#$ of pivot columns
$= 3$

$\dim(\text{null}) + \dim(\text{col}) =$ total # of columns

State bases for the null space and column space of A.

A basis for col(A) is $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -3 \\ -1 \end{bmatrix} \right\}$.

The nullspace comes from

$$\begin{cases} x_1 - 2x_3 - x_5 + 2x_6 = 0 \\ x_2 = 0 \\ x_4 - 3x_6 = 0 \end{cases}$$

So

$$\begin{cases} x_1 = 2x_3 + x_5 - 2x_6 \\ x_2 = 0 \\ x_4 = 3x_6 \end{cases}$$

$\therefore$ Basis for null(A) is $\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}$

Determine the dimension of span $\overbrace{\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ 4 \\ 0 \\ -16 \\ 8 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 11 \\ -6 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right\}}^{\text{Call the set } S}$. Explain your thought process!

$$\text{Let } A = \begin{bmatrix} 1 & -4 & -1 & 1 & -2 \\ -1 & 4 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 & 1 \\ 4 & -16 & 3 & 11 & -1 \\ -2 & 8 & -2 & -6 & 0 \end{bmatrix}$$

$$\text{col}(A) = \text{Span}(S)$$

$$A \sim \begin{bmatrix} 1 & -4 & 0 & 2 & -1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A has two pivot columns.

$\therefore \dim(\text{Span}(S)) = 2$

As a side note, this means that <u>any</u> two linearly independent vectors in S form a basis for S.

Find a basis for the null space of B where $B = \begin{bmatrix} -i & 1 \\ -1 & -i \end{bmatrix}$.

$$\begin{bmatrix} -i & 1 \\ -1 & -i \end{bmatrix} \xrightarrow{iR_1 + R_2 \to R_2} \begin{bmatrix} -i & 1 \\ 0 & 0 \end{bmatrix}$$

equation 2 was redundant

$$-ix_1 + x_2 = 0$$

$$\Rightarrow x_2 = ix_1$$

$$\text{Basis: } \left\{ \begin{bmatrix} i \\ -1 \end{bmatrix} \right\}$$