1. a. The statement is true. $\begin{bmatrix} a \\ b \\ 2a+b \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ so $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ obviously forms a basis for H. This means any two linearly independent vectors from H form a basis for H, and the vectors in $\left\{ \begin{bmatrix} 3 \\ 4 \\ 10 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix} \right\}$ meet that criteria.

   b. The statement is false. As shown in part (a), H is a two-dimensional space, so every basis of H contains exactly two vectors. Note: since all three vectors in $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix} \right\}$ are in H, it must be the case that the set is linearly dependent.

   c. The statement is false. The vector $\begin{bmatrix} 3 \\ 1 \\ 9 \end{bmatrix}$ is not in H, so it cannot possibly be in a basis for H!

2. a. The statement is self-evidently false. It takes at least five vectors to span $\mathbb{R}^5$ and there are only four columns in A.

   b. The statement is self-evidently true. The column space of A is a subspace of $\mathbb{R}^5$, and any vector in $\mathbb{R}^5$ can be written as a linear combination of $\vec{e}_1 - \vec{e}_5$.

   c. The statement is self-evidently false. The largest dimension that $\operatorname{col}(A)$ can possibly have is four (since A has only four columns), so there is no way a basis for $\operatorname{col}(A)$ can possibly contain five vectors.

   d. The statement is false. This is the first question in the bunch that requires you to find the reduced echelon form of A. From that form you see that A has only three pivot columns, so bases for $\operatorname{col}(A)$ contain exactly three vectors.; $\left\{ \vec{C}_1, \vec{C}_2, \vec{C}_3, \vec{C}_4 \right\}$ does not fit that bill.

   e. The statement is true. The vectors in $\left\{ \vec{C}_1, \vec{C}_2, \vec{C}_3 \right\}$ are the pivot columns of A.

f. The statement is false. The reduced echelon form of $\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 3 & 3 & 0 \\ -2 & 4 & -2 & 0 \\ 5 & 0 & 15 & 0 \\ 0 & 4 & 4 & 0 \end{array}\right]$ shows that $\left\{\vec{C}_1, \vec{C}_2, \vec{C}_4\right\}$ is a linearly dependent set, so the three vectors cannot possibly form a basis for anything.

g. The statement is true. The reduced echelon form of $\left[\begin{array}{ccc|c} -2 & 2 & 1 & 0 \\ 3 & 1 & 3 & 0 \\ 4 & 1 & -2 & 0 \\ 0 & 0 & 15 & 0 \\ 4 & 0 & 4 & 0 \end{array}\right]$ shows that $\left\{\vec{C}_2, \vec{C}_3, \vec{C}_4\right\}$ is a linearly independent set. Since the dimension of $\operatorname{col}(A)$ is three, any three linearly independent vectors from $\operatorname{col}(A)$ form a basis for $\operatorname{col}(A)$.

h. The statement is false. The null space of A is a subspace of $\mathbb{R}^4$. (The null space consists of solutions to $A\vec{x} = \vec{0}$; the multiplication doesn't work unless $\vec{x}$ is 4×1.)

i. The statement is false. Again, the multiplication doesn't work unless $\vec{x}$ is 4×1, so the domain of T is $\mathbb{R}^4$.

j. The statement is true. The product $A\vec{x}$ is 5×1, so the images produced by T lie in $\mathbb{R}^5$ which means that the codomain of T is all of $\mathbb{R}^5$.

k. The statement is false. The reduced echelon form of $\left[\begin{array}{cccc|c} 1 & -2 & 2 & 1 & 0 \\ 0 & 3 & 1 & 3 & 0 \\ -2 & 4 & 1 & -2 & 0 \\ 5 & 0 & 0 & 15 & 0 \\ 0 & 4 & 0 & 4 & 1 \end{array}\right]$ shows that there are no solutions to the equation $A\vec{x} = \vec{e}_5$.