

1. Consider the subspace of \mathbb{R}^3 consisting of vectors of form $\begin{bmatrix} a \\ b \\ 2a+b \end{bmatrix}$; call the set H .

a. True or false? The vectors in $\left\{ \begin{bmatrix} 3 \\ 4 \\ 10 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix} \right\}$ form a basis for H ? Explain!

b. True or false? The vectors in $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix} \right\}$ form a basis for H ? Explain!

c. True or false? The vectors in $\left\{ \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 9 \end{bmatrix} \right\}$ form a basis for H ? Explain!

2. Consider $A = \begin{bmatrix} 1 & -2 & 2 & 1 \\ 0 & 3 & 1 & 3 \\ -2 & 4 & 1 & -2 \\ 5 & 0 & 0 & 15 \\ 0 & 4 & 0 & 4 \end{bmatrix}$. Let's call the columns of the matrix $\vec{C}_1, \vec{C}_2, \vec{C}_3, \vec{C}_4$.

- True or false? The column space of A is all of \mathbb{R}^5 ? Explain!
- True or false? Every vector from the column space of A can be written as a linear combination of the vectors $\vec{e}_1 - \vec{e}_5$ from \mathbb{R}^5 ? Explain!
- True or false? The vectors $\vec{e}_1 - \vec{e}_5$ from \mathbb{R}^5 form a basis for $\text{col}(A)$? Explain!
- True or false? The vectors in $\{\vec{C}_1, \vec{C}_2, \vec{C}_3, \vec{C}_4\}$ form a basis for $\text{col}(A)$? Explain!
- True or false? The vectors in $\{\vec{C}_1, \vec{C}_2, \vec{C}_3\}$ form a basis for $\text{col}(A)$? Explain!
- True or false? The vectors in $\{\vec{C}_1, \vec{C}_2, \vec{C}_4\}$ form a basis for $\text{col}(A)$? Explain!
- True or false? The vectors in $\{\vec{C}_2, \vec{C}_3, \vec{C}_4\}$ form a basis for $\text{col}(A)$? Explain!
- True or false? The null space of A is a subspace of \mathbb{R}^5 ? Explain!
- Consider the linear transformation $T(\vec{x}) = A\vec{x}$. True or false? The domain of T is \mathbb{R}^5 ? Explain!
- True or false? The vector \vec{e}_5 from \mathbb{R}^5 is in the codomain of T ? Explain!
- True or false? The vector \vec{e}_5 from \mathbb{R}^5 is in the range of T ? Explain!