Definition

The ***kernel*** of the linear transformation T is the set of all solutions to the equation $T(\vec{x}) = \vec{0}$.

Example

Suppose that $T:\ \mathbb{R}^3 \to \mathbb{R}^4$ and that $T(\vec{e}_1) = \begin{bmatrix} 2 & -4 & 1 & 2 \end{bmatrix}^{\mathrm{T}}$, $T(\vec{e}_2) = \begin{bmatrix} -1 & 2 & -2 & -7 \end{bmatrix}^{\mathrm{T}}$, and $T(\vec{e}_3) = \begin{bmatrix} 3 & -6 & 1 & 1 \end{bmatrix}$. Find the kernel and range of T.

Example

Suppose that $T:\ P_2 \to \mathbb{R}^2$ with the rule $T(\vec{p}) = \begin{bmatrix} \vec{p}(3) \\ \vec{p}(5) \end{bmatrix}$. What is the image of $\vec{p}_1$ where $\vec{p}_1(t) = 2 - t + 2t^2$? What do polynomials in the kernel of T all have in common? Find the transformation matrix for T using (from $\mathbb{R}^3$) the vectors $\vec{e}_1$, $\vec{e}_2$, and $\vec{e}_3$ to represent, respectively, the polynomials with formulas "1," "t," and "t^2". Find the null space of the transformation matrix and relate it back to polynomials in the kernel of T.