

We're going to start with a couple of background theorems. Let's prove each of the following.

1. Prove that if  $T$  is a linear transformation, then  $T(\vec{0}) = \vec{0}$ .
- b. Prove that the linear transformation  $T$  is one-to-one if and only if the only solution to  $T(\vec{x}) = \vec{0}$  is  $\vec{0}$ . **Hint:** Prove the **contrapositive statement**.

Find all values of  $\lambda$  that create non-trivial solutions to the system  $A\vec{x} = \vec{0}$  where  $A$  is the matrix given below. Make sure that you show all relevant work and that both your reasoning and your conclusion are clear.

$$A = \begin{bmatrix} 1 & \lambda & 0 \\ 1 & 3 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$