We're going to start with a couple of background theorems. Let's prove each of the following.

1. Prove that if T is a linear transformation, then $T\left(\vec{0}\right) = \vec{0}$.

b. Prove that the linear transformation T is one-to-one if and only if the only solution to $T\left(\vec{x}\right) = \vec{0}$ is $\vec{0}$. **Hint**: Prove the ***contrapositive statement***.

1. Since T is a linear transformation,

we know that $T(\vec{0} + \vec{x}) = T(\vec{0}) + T(\vec{x})$

$\therefore \quad T(\vec{x}) = T(\vec{0}) + T(\vec{x})$

$\therefore \quad T(\vec{0}) = T(\vec{x}) - T(\vec{x})$
$= \vec{0}$

b. Contrapositive statement

T is not one-to-one iff $T(\vec{x}) = \vec{0}$ has non-zero vector solutions.

T is not one-to-one $\iff \exists\, \vec{u}$ and $\vec{v}$, $\vec{u} \neq \vec{v}$, $\ni T(\vec{u}) = T(\vec{v})$

$\iff \exists\, \vec{u}$ and $\vec{v}$, $\vec{u} \neq \vec{v}$, $\ni T(\vec{u}) - T(\vec{v}) = \vec{0}$

$\iff \exists\, \vec{u}$ and $\vec{v}$, $\vec{u} \neq \vec{v}$, $\ni T(\vec{u} - \vec{v}) = \vec{0}$

QED

(If $\vec{u} \neq \vec{v}$, $\vec{u} - \vec{v} \neq \vec{0}$)

MTH 261 – Mr. Simonds' class

Find all values of λ that create non-trivial solutions to the system $A\vec{x} = \vec{0}$ where A is the matrix given below. Make sure that you show all relevant work and that both your reasoning and your conclusion are clear.

$$A = \begin{bmatrix} 1 & \lambda & 0 \\ 1 & 3 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

The system $A\vec{x} = \vec{0}$ will have non-trivial solutions iff $\det(A) = 0$ (properties d & m from "Theorem 8.")

$$\det(A) = \sum_{i=1}^{3} \left[a_{i,3} C_{i,3} \right]$$

$$= a_{13} C_{13} + a_{23} C_{23} + a_{33} C_{33}$$

$$= 0 + (1)(-1)^5 \begin{vmatrix} 1 & \lambda \\ 2 & 1 \end{vmatrix} + 1(-1)^6 \begin{vmatrix} 1 & \lambda \\ 1 & 3 \end{vmatrix}$$

$$= (-1)(1 - 2\lambda) + (3 - \lambda)$$

$$= 2 + \lambda$$

$\det(A) = 0 \implies \lambda = -2$

$\therefore$ $A\vec{x} = \vec{0}$ has non-trivial solutions iff $\lambda = -2$