

Several linear transformations are stated on below and on the next page. For each transformation, determine if the transformation is bijective, only injective, only surjective, or nojective. Then:

- If the transformation is not injective, determine an explicit, non-zero vector  $\vec{x}$  that maps to  $\vec{0}$ . Show work that leads to the vector. That work might simply be a statement that begins "(i)t is obvious that ...". Finally, use the stated pattern to explicitly show that  $\vec{x}$  maps to  $\vec{0}$ .
- If the transformation is not surjective, determine an explicit vector in the codomain that is not in the range of the transformation. Show work that leads to the vector. That work might simply be a statement that begins "(i)t is obvious that ...".

Linear Transformation 1:  $T_1\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -2x_1 + x_3 \\ x_1 + x_2 \\ 3x_1 + x_2 - x_3 \end{bmatrix}^T$

Linear Transformation 2:  $T_2\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3x_2 \\ -x_3 \\ 4x_1 + x_3 \end{bmatrix}^T$

Linear Transformation 3:  $T_3\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T\right) = \begin{bmatrix} x_2 \\ 0 \\ x_1 - x_2 \end{bmatrix}^T$

Linear Transformation 4:  $T_4\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}^T\right) = \begin{bmatrix} 2x_1 + x_2 - x_4 \\ x_1 + x_2 - x_3 + 2x_4 \\ -x_2 + 2x_3 - 5x_4 \end{bmatrix}$