

- Suppose that $\vec{v}_3 = 2\vec{v}_1 - 4\vec{v}_2$. Why does this imply that the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent?
- Suppose that $\vec{v}_4 = -\vec{v}_1$. Why does this imply that the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is linearly dependent?
- Suppose that $\vec{v}_2 = \vec{0}$. Why does this imply that the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent?
- Let $\vec{v}_1 = \begin{bmatrix} a \\ b \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} c \\ d \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} e \\ f \end{bmatrix}$ (none of which are zero vectors). Use a matrix to establish that there is no way that the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent. **Hint:** Consider the reduced row echelon form of the matrix $\begin{bmatrix} a & c & e & | & 0 \\ b & d & f & | & 0 \end{bmatrix}$. There are only three possibilities for the RREF matrix (treating non-mandated entries as arbitrary numbers). What do each of the possibilities imply about the equation $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 = \vec{0}$?

- Suppose that the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is linearly dependent. Explain why this implies that at least one of the vectors in the set can be written as a linear combination of the remaining vectors in the set.
- Suppose that the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is linearly independent. Explain why this implies that none of the vectors in the set can be written as a linear combination of the remaining vectors in the set.
- Suppose that the set $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent and that \vec{v}_3 is not in the span of $\{\vec{v}_1, \vec{v}_2\}$. Explain why this means that the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ must also be linearly independent.
- Two linearly independent vectors from \mathbb{R}^2 are shown in Figure 1. In each of figures 2-4 a third vector has been introduced. Express each of the new vectors as linear combinations of \vec{x}_1 and \vec{x}_2 . Please note that all of the weights are integers.

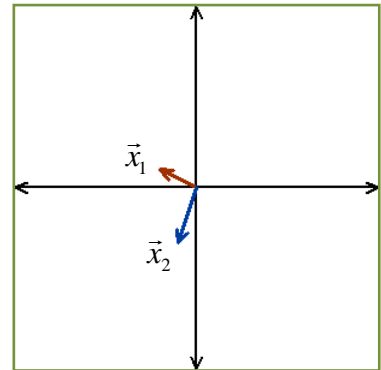


Figure 1

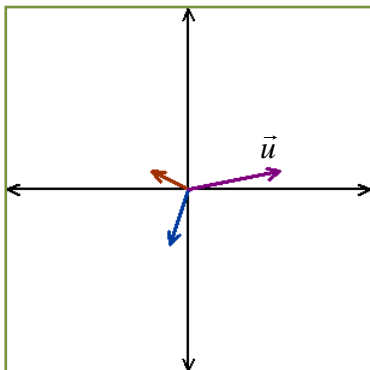


Figure 2

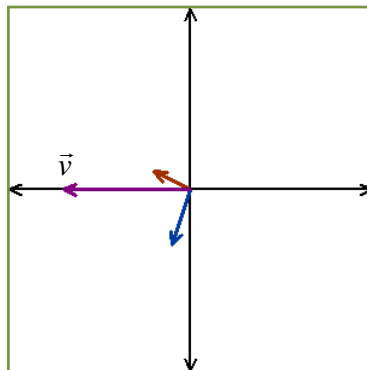


Figure 3

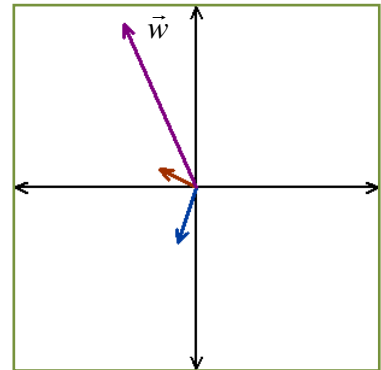


Figure 4