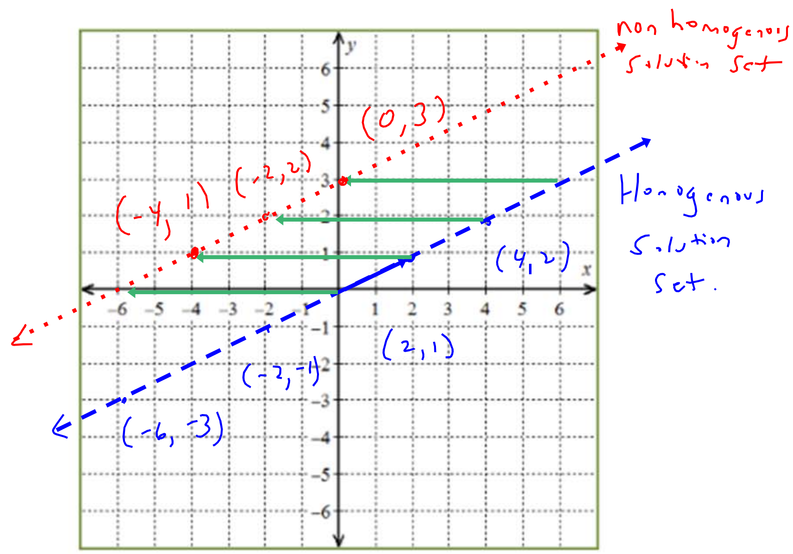
Compare and contrast the solution set to the homogenous system $\begin{cases} x_1 - 2x_2 = 0 \\ -2x_1 + 4x_2 = 0 \end{cases}$ with the solution set to the nonhomogeneous system $\begin{cases} x_1 - 2x_2 = -6 \\ -2x_1 + 4x_2 = 12 \end{cases}$. State all solution sets as vector equations and state the solution set to the nonhomogeneous system twice, the second time changing the shift vector relative to the first statement of the solution set. Finally, graph the solution sets and illustrate the shifting effect between the solution sets.

$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ -2 & 4 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

So, $x_1 = 2x_2$, so solutions have form

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$



$$\left[\begin{array}{cc|c} 1 & -2 & -6 \\ -2 & 4 & 12 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & -6 \\ 0 & 0 & 0 \end{array}\right]$$

So, $x_1 = 2x_2 - 6$, so solutions have form

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_2 - 6 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -6 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} -6 \\ 0 \end{bmatrix}$ is the shift vector your text calls $\vec{p}$.

I can use <u>any</u> specific solution to the nonhomogenous system as the shift vector.

eg. I could state the nonhomogenous solution as

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$