

$$\vec{u}_1 \cdot \vec{u}_2 = -2 + 1 + 1 = 0$$

$$1. \quad \vec{u}_1 \cdot \vec{u}_2 = 0 - 1 + 1 = 0$$

$$\vec{u}_2 \cdot \vec{u}_3 = 0 - 1 + 1 = 0$$

$$\text{comp}_{\vec{u}_1}(\vec{v}) = \frac{\vec{v} \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} = \frac{2}{3}, \quad \text{comp}_{\vec{u}_2}(\vec{v}) = \frac{\vec{v} \cdot \vec{u}_2}{\vec{u}_2 \cdot \vec{u}_2} = \frac{-4}{6} = -\frac{2}{3}, \quad \text{comp}_{\vec{u}_3}(\vec{v}) = \frac{\vec{v} \cdot \vec{u}_3}{\vec{u}_3 \cdot \vec{u}_3} = \frac{-2}{2} = -1$$

$$\text{Check: } \begin{pmatrix} 2 \\ 3 \end{pmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} + (-1) \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \vec{v}$$

$$2. \quad \vec{w}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{w}_2 = \vec{v}_2 - \frac{\vec{v}_2 \cdot \vec{w}_1}{\vec{w}_1 \cdot \vec{w}_1} \vec{w}_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} - \left(\frac{1}{4}\right) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -5/4 \\ 3/4 \\ 3/4 \\ -1/4 \end{bmatrix}. \quad \text{Rename } \vec{w}_2 = \begin{bmatrix} -5 \\ 3 \\ 3 \\ -1 \end{bmatrix}.$$

$$\vec{w}_3 = \vec{v}_3 - \frac{\vec{v}_3 \cdot \vec{w}_1}{\vec{w}_1 \cdot \vec{w}_1} \vec{w}_1 - \frac{\vec{v}_3 \cdot \vec{w}_2}{\vec{w}_2 \cdot \vec{w}_2} \vec{w}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix} - \left(\frac{2}{4}\right) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \left(\frac{6}{44}\right) \begin{bmatrix} -5 \\ 3 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -9/11 \\ -10/11 \\ 1/11 \\ 18/11 \end{bmatrix}. \quad \text{Rename } \vec{w}_3 = \begin{bmatrix} -9 \\ -10 \\ 1 \\ 18 \end{bmatrix}$$

$$\text{Our orthonormal basis is } \left\{ \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{44}} \begin{bmatrix} -5 \\ 3 \\ 3 \\ -1 \end{bmatrix}, \frac{1}{\sqrt{506}} \begin{bmatrix} -9 \\ -10 \\ 1 \\ 18 \end{bmatrix} \right\}. \quad \text{Call the vectors in this set,}$$

respectively,  $\hat{w}_1$ ,  $\hat{w}_2$ , and  $\hat{w}_3$ . Then:

$$\text{comp}_{\hat{w}_1}(\vec{v}) = \vec{v} \cdot \hat{w}_1 = 13, \quad \text{comp}_{\hat{w}_2}(\vec{v}) = \vec{v} \cdot \hat{w}_2 = \frac{14}{\sqrt{44}}, \quad \text{comp}_{\hat{w}_3}(\vec{v}) = \vec{v} \cdot \hat{w}_3 = \frac{276}{\sqrt{506}}$$

$$\text{Check: } (13) \left(\frac{1}{2}\right) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \left(\frac{14}{\sqrt{44}}\right) \left(\frac{1}{\sqrt{44}}\right) \begin{bmatrix} -5 \\ 3 \\ 3 \\ -1 \end{bmatrix} + \left(\frac{276}{\sqrt{506}}\right) \left(\frac{1}{\sqrt{506}}\right) \begin{bmatrix} -9 \\ -10 \\ 1 \\ 18 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 8 \\ 16 \end{bmatrix} = \vec{v}$$