Introductory Example

Use an augmented matrix to mimic the elimination method to solve the linear system of equations $\begin{cases} 2x_1 - 3x_2 = 8 \\ 6x_1 + x_2 = -36 \end{cases}$.

Example

Use the method of Gaussian elimination to find an echelon form of the augmented matrix representation for each of the following systems of equations and use that matrix to determine the solution to the system of equations.

a. $\begin{cases} 2x_1 - 5x_2 - 3x_3 = -23 \\ -5x_1 + x_2 - 2x_3 = -7 \\ x_1 + 3x_2 + x_3 = 3 \end{cases}$

b.
$$\begin{aligned} x_1 - 2x_2 + x_4 &= 4 \\ -2x_1 + 3x_2 - 2x_3 + 2x_4 &= -3 \\ - x_2 - 2x_3 &= -11 \\ 5x_1 - 10x_2 - 3x_4 &= -1 \end{aligned}$$

What does the echelon form of the matrix in part (b) tell you about the solution set to the system of equations modeled by the matrix stated in part (b)?

Example

Several augmented row echelon form matrices are given below (and on the next page). For each matrix, identify the pivot columns and state the nature of the solution set for the associated system of equations.

a. $\mathbf{A} = \left[\begin{array}{ccc|c} 2 & 5 & 5 & -2 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

b. $\mathbf{B} = \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 8 & 16 \\ 0 & 0 & 0 \end{array}\right]$

c. $\mathbf{C} = \left[\begin{array}{cccc|c} -2 & 1 & -1 & 6 & 0 \\ 0 & 4 & -1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

Example

Solve the next three systems using the Gauss-Jordan elimination method. $\begin{cases} 2x_1 + 3x_2 = -2 \\ 6x_1 - 6x_2 = -1 \end{cases}$

$$\begin{cases} 2x_2 - 6x_3 = -2 \\ 4x_1 - x_2 + 3x_3 = 1 \\ -x_1 + 3x_2 - 8x_3 = -4 \end{cases}$$

$$\begin{cases} -x_1 + 6x_2 - 2x_3 = 9 \\ 3x_1 - 2x_2 + x_3 + 5x_4 = -1 \\ 2x_1 + 4x_2 - x_3 + 5x_4 = 8 \\ -3x_1 - x_2 + x_3 - 7x_4 = -6 \end{cases}$$

Example
State the general solution to the following system of equations and rigorously verify the solution. Also, state two specific solutions to the system.

$$\begin{cases} x_1 - 2x_2 + 2x_3 - 3x_4 = 2 \\ \quad x_2 - x_3 + 2x_4 = -3 \end{cases}$$

Example
State a ***general solution*** to the following system of equations as well as two ***specific solutions*** to the system.

$$\begin{cases} 2x_1 + 6x_2 + 5x_3 = -2 \\ -x_1 - 3x_2 + 3x_3 = 1 \end{cases}$$

Example

Is it possible to find a value of C that makes the following system inconsistent? How do you know?

$$\begin{cases} 2x_2 + x_3 = -2 \\ x_1 - 4x_2 - 4x_3 = C \\ 2x_1 + 12x_2 + 5x_3 = 7 \end{cases}$$

Example

Under what conditions is the following system of equations consistent?

$$\begin{cases} x_1 + x_2 + 2x_3 = a \\ x_1 + 3x_2 - x_3 = b \\ -2x_1 - 8x_2 + 5x_3 = c \end{cases}$$

Application: Balancing Chemical Equations

Ethane and Oxygen combine to produce Carbon Dioxide and steam. Formally, this is represented by the equation $C_2H_6 + O_2 \rightarrow CO_2 + H_2O$. Let's put our newfound skills to use and balance this equation. Speaking of putting things to use … let's use our calculator to find the RREF form of the matrix.

Application: Network Analysis

A network is most easily thought of as a city street system. The intersections are technically called ***nodes*** or ***junctions*** and each directed stretch of road between intersections is called a ***branch***. Because branches are directed, if there is a two-way street between two intersections the corresponding network will have two branches between the corresponding nodes.

We assign values or variables to each branch; those values and variables could conceptually represent flow-rates or flow-amounts along those branches. In order for the network to be valid, **the total flow into the network must equal the total flow out of the network**.

The values and variables in Figure 1 represent traffic flow rates (vehicles/quarter-hour) in a small section of a city street system. Let's determine the minimum and maximum flow rates through each of the variable branches.

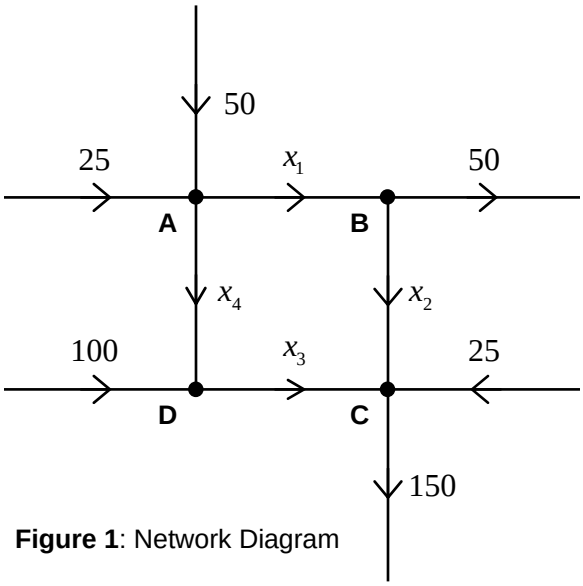

Figure 1: Network Diagram