Application: Balancing Chemical Equations

Ethane and Oxygen combine to produce Carbon Dioxide and steam. Formally, this is represented by the equation $C_2H_6 + O_2 \rightarrow CO_2 + H_2O$. Let's put our newfound skills to use and balance this equation. Speaking of putting things to use … let's use our calculator to find the RREF form of the matrix.

Define: x_1 as the number of C_2H_6 molecules in the mixture;
x_2 as the number of O_2 molecules in the mixture;
x_3 as the number of resultant CO_2 molecules;
x_4 as the number of resultant H_2O molecules

Balance Equations

C: $2x_1 = 1x_3$

H: $6x_1 = 2x_4$

O: $2x_2 = 2x_3 + x_4$

System

$$\begin{cases} 2x_1 \qquad - x_3 \qquad = 0 \\ 6x_1 \qquad\qquad - 2x_4 = 0 \\ \qquad 2x_2 - 2x_3 - x_4 = 0 \end{cases}$$

$$\left[\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 6 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -7/6 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \end{array}\right]$$

general solution

$$\begin{cases} x_1 = \frac{1}{3}x_4 \\ x_2 = \frac{7}{6}x_4 \\ x_3 = \frac{2}{3}x_4 \\ x_4 = \text{free} \end{cases}$$

But, there ain't no such thing as a fractional number of atoms.

$\therefore$ The system is in balance when $x_4 = 6$.

Balanced Eq: $2C_2H_6 + 7O_2 \rightarrow 4CO_2 + 6H_2O$

Application: Network Analysis

A network is most easily thought of as a city street system. The intersections are technically called ***nodes*** or ***junctions*** and each directed stretch of road between intersections is called a ***branch***. Because branches are directed, if there is a two-way street between two intersections the corresponding network will have two branches between the corresponding nodes.

We assign values or variables to each branch; those values and variables could conceptually represent flow-rates or flow-amounts along those branches. In order for the network to be valid, **the total flow into the network must equal the total flow out of the network**.

The values and variables in Figure 1 represent traffic flow rates (vehicles/quarter-hour) in a small section of a city street system. Let's determine the minimum and maximum flow rates through each of the variable branches.

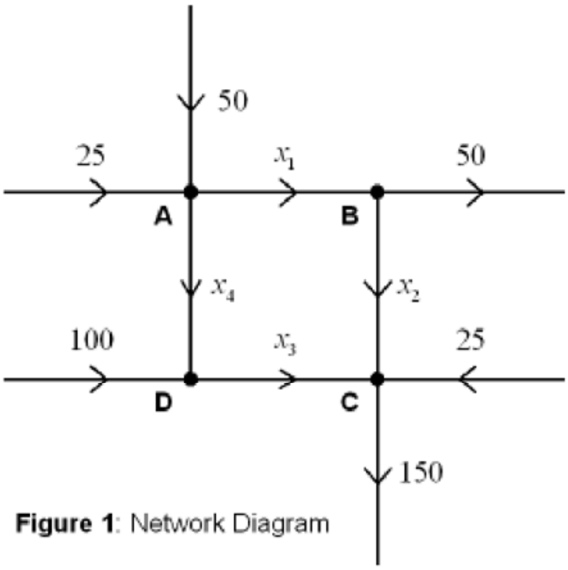

Figure 1: Network Diagram

Inflow = outflow

A : $75 = x_1 + x_4$

B : $x_1 = 50 + x_2$

C : $x_2 + x_3 + 25 = 150$

D : $x_4 + 100 = x_3$

System

$$\begin{cases} x_1 + x_4 = 75 \\ x_1 - x_2 = 50 \\ x_2 + x_3 = 125 \\ -x_3 + x_4 = -100 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 75 \\ 1 & -1 & 0 & 0 & 50 \\ 0 & 1 & 1 & 0 & 125 \\ 0 & 0 & -1 & 1 & -100 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 75 \\ 0 & 1 & 0 & 1 & 25 \\ 0 & 0 & 1 & -1 & 100 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

General Solution

$$\begin{cases} x_1 = -x_4 + 75 \\ x_2 = -x_4 + 25 \\ x_3 = x_4 + 100 \\ x_4 = \text{free} \end{cases}$$

Since there can't be a negative number of cars

$0 \leq x_4 \leq 25$ ← x_2 can't be negative

$\therefore$

$50 \leq x_1 \leq 75$

$0 \leq x_2 \leq 25$

$100 \leq x_3 \leq 125$

$0 \leq x_4 \leq 25$