Application: Balancing Chemical Equations

Ethane and Oxygen combine to produce Carbon Dioxide and steam. Formally, this is represented by the equation $C_2H_6 + O_2 \to CO_2 + H_2O$. Let's put our newfound skills to use and balance this equation. Speaking of putting things to use … let's use our calculator to find the RREF form of the matrix.

Let's model the molecule thus: $\begin{bmatrix} \text{\# of carbon atoms} \\ \text{\# of hydrogen atoms} \\ \text{\# of oxygen atoms} \end{bmatrix}$

Let x_1 be the number of C_2H_6 molecules,
x_2 be the number of O_2 molecules
x_3 be the number of CO_2 molecules
and x_4 be the number of H_2O molecules

$$x_1\begin{bmatrix}2\\6\\0\end{bmatrix} + x_2\begin{pmatrix}0\\0\\2\end{pmatrix} = x_3\begin{bmatrix}1\\0\\2\end{bmatrix} + x_4\begin{bmatrix}0\\2\\1\end{bmatrix}$$

$$\begin{cases} 2x_1 = x_3 \\ 6x_1 = 2x_4 \\ 2x_2 = 2x_3 + x_4 \end{cases} \Rightarrow \begin{cases} 2x_1 - x_3 = 0 \\ 6x_1 - 2x_4 = 0 \\ 2x_2 - 2x_3 - x_4 = 0 \end{cases}$$

$$\left(\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 6 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -7/6 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \end{array}\right)$$

General solution: $\begin{cases} x_1 = \frac{1}{3}x_4 \\ x_2 = \frac{7}{6}x_4 \\ x_3 = \frac{2}{3}x_4 \\ x_4 \text{ is free} \end{cases}$

We need to x_4 value so that all other values are positive integers

Let $x_4 = 6 \Rightarrow x_1 = 2,\ x_2 = 7,\ x_3 = 4$

$\therefore\ 2C_2H_6 + 7O_2 \to 4CO_2 + 6H_2O$

Application: Network Analysis

A network is most easily thought of as a city street system. The intersections are technically called ***nodes*** or ***junctions*** and each directed stretch of road between intersections is called a ***branch***. Because branches are directed, if there is a two-way street between two intersections the corresponding network will have two branches between the corresponding nodes.

We assign values or variables to each branch; those values and variables could conceptually represent flow-rates or flow-amounts along those branches. In order for the network to be valid, **the total flow into the network must equal the total flow out of the network**.

The values and variables in Figure 1 represent traffic flow rates (vehicles/quarter-hour) in a small section of a city street system. Let's determine the minimum and maximum flow rates through each of the variable branches.

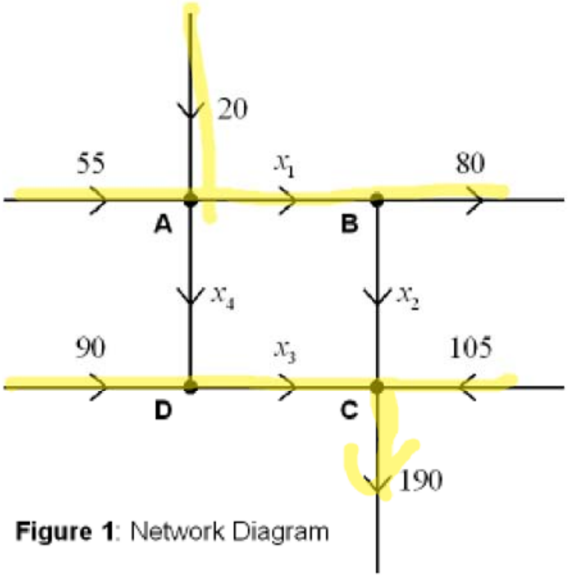

Figure 1: Network Diagram

nodal Equations

A: $20+55 = x_1+x_4$

B: $x_1 = x_2+80$

C: $x_2+x_3+105=190$

D: $90+x_4 = x_3$

$$\left[\begin{array}{cccc|c} -1 & 0 & 0 & -1 & -75 \\ 1 & -1 & 0 & 0 & 80 \\ 0 & 1 & 1 & 0 & 85 \\ 0 & 0 & -1 & 1 & -90 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 75 \\ 0 & 1 & 0 & 1 & -5 \\ 0 & 0 & 1 & -1 & 90 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

General solution: $\begin{cases} x_1 = -x_4+75 \\ x_2 = -x_4-5 \\ x_3 = x_4+90 \\ x_4 \text{ is free} \end{cases}$

All flow rates must non-negative.

x_4 and x_2 cannot both be non-negative.

This network is impossible.