1. $\left[\begin{array}{cc|c} 0 & 2 & 10 \\ 2 & 5 & 21 \end{array}\right] \xrightarrow{R1 \leftrightarrow R2} \left[\begin{array}{cc|c} 2 & 5 & 21 \\ 0 & 2 & 10 \end{array}\right]$

$\xrightarrow{\frac{1}{2}R2 \to R2} \left[\begin{array}{cc|c} 2 & 5 & 21 \\ 0 & 1 & 5 \end{array}\right]$

$\xrightarrow{-5R2 + R1 \to R1} \left[\begin{array}{cc|c} 2 & 0 & -4 \\ 0 & 1 & 5 \end{array}\right]$

$\xrightarrow{\frac{1}{2}R1 \to R1} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 5 \end{array}\right]$

3. $\left[\begin{array}{ccc|c} -4 & 3 & 5 & -22 \\ -1 & 3 & 8 & -10 \\ 6 & 2 & 12 & 20 \end{array}\right] \xrightarrow[\frac{1}{2}R3 \to R3]{R1 \leftrightarrow R2} \left[\begin{array}{ccc|c} -1 & 3 & 8 & -10 \\ -4 & 3 & 5 & -22 \\ 3 & 1 & 6 & 10 \end{array}\right]$

$\xrightarrow[3R1 + R3 \to R3]{-4R1 + R2 \to R2} \left[\begin{array}{ccc|c} -1 & 3 & 8 & -10 \\ 0 & -9 & -27 & 18 \\ 0 & 10 & 30 & -20 \end{array}\right]$

$\xrightarrow[\frac{1}{10}R3 \to R3]{-\frac{1}{9}R2 \to R2} \left[\begin{array}{ccc|c} -1 & 3 & 8 & -10 \\ 0 & 1 & 3 & -2 \\ 0 & 1 & 3 & -2 \end{array}\right]$

$\xrightarrow[-R2 + R3 \to R3]{-3R2 + R1 \to R1} \left[\begin{array}{ccc|c} -1 & 0 & -1 & -4 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\xrightarrow{-R1 \to R1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

5. $\left[\begin{array}{ccc|c} 3 & 4 & -9 & 14 \\ -5 & 2 & 15 & 20 \\ 1 & 1 & -3 & 3 \end{array}\right] \xrightarrow{R1 \leftrightarrow R3} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 3 \\ -5 & 2 & 15 & 20 \\ 3 & 4 & -9 & 14 \end{array}\right]$

$\xrightarrow[-3R1 + R3 \to R3]{5R1 + R2 \to R2} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 3 \\ 0 & 7 & 0 & 35 \\ 0 & 1 & 0 & 5 \end{array}\right]$

$\xrightarrow{\frac{1}{7}R2 \to R2} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 1 & 0 & 5 \end{array}\right]$

$\xrightarrow[-R2 + R3 \to R3]{-R2 + R1 \to R1} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$

7.
$$\left[\begin{array}{cccc|c} 0 & 3 & 5 & 1 & -7 \\ 1 & 1 & 4 & -2 & -5 \\ 4 & 3 & -1 & -1 & 5 \\ 6 & 1 & 1 & 3 & 13 \end{array}\right] \xrightarrow{R1 \leftrightarrow R2} \left[\begin{array}{cccc|c} 1 & 1 & 4 & -2 & -5 \\ 0 & 3 & 5 & 1 & -7 \\ 4 & 3 & -1 & -1 & 5 \\ 6 & 1 & 1 & 3 & 13 \end{array}\right]$$

$$\xrightarrow[-6R1 + R4 \to R4]{-4R1 + R3 \to R3} \left[\begin{array}{cccc|c} 1 & 1 & 4 & -2 & -5 \\ 0 & 3 & 5 & 1 & -7 \\ 0 & -1 & -17 & 7 & 25 \\ 0 & -5 & -23 & 15 & 43 \end{array}\right]$$

$$\xrightarrow{R2 \leftrightarrow R3} \left[\begin{array}{cccc|c} 1 & 1 & 4 & -2 & -5 \\ 0 & -1 & -17 & 7 & 25 \\ 0 & 3 & 5 & 1 & -7 \\ 0 & -5 & -23 & 15 & 43 \end{array}\right]$$

$$\xrightarrow[-5R2 + R4 \to R4]{3R2 + R3 \to R3} \left[\begin{array}{cccc|c} 1 & 1 & 4 & -2 & -5 \\ 0 & -1 & -17 & 7 & 25 \\ 0 & 0 & -46 & 22 & 68 \\ 0 & 0 & 62 & -20 & -82 \end{array}\right]$$

$$\xrightarrow{\frac{62}{46}R3 + R4 \to R4} \left[\begin{array}{cccc|c} 1 & 1 & 4 & -2 & -5 \\ 0 & -1 & -17 & 7 & 25 \\ 0 & 0 & -46 & 22 & 68 \\ 0 & 0 & 0 & 222/23 & 222/23 \end{array}\right]$$

$$\xrightarrow{\frac{23}{222}R4 \to R4} \left[\begin{array}{cccc|c} 1 & 1 & 4 & -2 & -5 \\ 0 & -1 & -17 & 7 & 25 \\ 0 & 0 & -46 & 22 & 68 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$\xrightarrow[-22R4 + R3 \to R3]{2R4 + R1 \to R1 \\ -7R4 + R2 \to R2} \left[\begin{array}{cccc|c} 1 & 1 & 4 & 0 & -3 \\ 0 & -1 & -17 & 0 & 18 \\ 0 & 0 & -46 & 0 & 46 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{46}R3 \to R3} \left[\begin{array}{cccc|c} 1 & 1 & 4 & 0 & -3 \\ 0 & -1 & -17 & 0 & 18 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$\xrightarrow[17R3 + R2 \to R2]{-4R3 + R1 \to R1} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{-R2 \to R2} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{-R2 + R1 \to R1} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

1. The solution to $\begin{cases} 2x_2 = 10 \\ 2x_1 + 5x_2 = 21 \end{cases}$ is $(-2,5)$.

2. The solution to $\begin{cases} 2x_1 - 3x_2 = 17 \\ 5x_1 + 2x_2 = 14 \end{cases}$ is $(4,-3)$.

3. The general solution to $\begin{cases} -4x_1 + 3x_2 + 5x_3 = -22 \\ -x_1 + 3x_2 + 8x_3 = -10 \\ 6x_1 + 2x_2 + 12x_3 = 20 \end{cases}$ is $\begin{cases} x_1 = -x_3 + 4 \\ x_2 = -3x_3 - 2 \\ x_3 \text{ is free} \end{cases}$.

4. The solution to $\begin{cases} 4x_2 + 2x_3 = -4 \\ 2x_1 - x_2 - x_3 = 0 \\ 5x_1 + 7x_2 = -23 \end{cases}$ is $(1,-4,6)$.

5. The general solution to $\begin{cases} 3x_1 + 4x_2 - 9x_3 = 14 \\ -5x_1 + 2x_2 + 15x_3 = 20 \\ x_1 + x_2 - 3x_3 = 3 \end{cases}$ is $\begin{cases} x_1 = 3x_3 - 2 \\ x_2 = 5 \\ x_3 \text{ is free} \end{cases}$.

6. There are no solutions to $\begin{cases} x_1 - 2x_2 + x_3 = 2 \\ -2x_1 + 4x_2 + 3x_3 = -7 \\ 6x_1 - 12x_2 - 7x_3 = 10 \end{cases}$.

7. The solution to $\begin{cases} 3x_2 + 5x_3 + x_4 = -7 \\ x_1 + x_2 + 4x_3 - 2x_4 = -5 \\ 4x_1 + 3x_2 - x_3 - x_4 = 5 \\ 6x_1 + x_2 + x_3 + 3x_4 = 13 \end{cases}$ is $(2,-1,-1,1)$.

8. The general solution to $\begin{cases} x_1 + 3x_2 + x_3 + 3x_4 = 6 \\ x_1 - x_2 - x_3 - 13x_4 = -2 \\ -2x_1 - 3x_2 + x_3 + 6x_4 = -3 \\ 3x_1 + 7x_2 + 2x_3 + x_4 = 14 \end{cases}$ is $\begin{cases} x_1 = 9x_4 + 1 \\ x_2 = -4x_4 + 1 \\ x_3 = 2 \\ x_4 \text{ is free} \end{cases}$.