The Gaussian elimination process for solving systems of linear equations is predicated upon the three ***elementary row operations***. The three elementary row operations are:

1. Interchanging two rows of the matrix
2. Replacing one row of the matrix with a non-zero multiple of itself
3. Replacing one row of a matrix with itself added to a multiple of another row of the matrix

Two matrices are said to be ***row equivalent*** if one matrix can be transformed into the other via a series of elementary row operations. The first non-zero entry of any row of a matrix is called ***the leading entry*** of that row. A matrix is said to be in **(*row*) *echelon form*** if the matrix satisfies both of the following properties.

i. Every row that contains nothing but zeros occurs at the bottom of the matrix.
ii. The leading entry of any non-zero row appears to the right of the leading entry in the row directly above it.

The process of transforming a matrix into a row equivalent matrix of echelon form is called the ***Gaussian elimination process***.

Definitions and Fundamental Facts

A ***pivot position*** in a matrix is an entry position that corresponds to a leading entry in a row echelon form of the matrix. Any column that contains a pivot entry is called a ***pivot column***.

If A is an augmented matrix representing a system of equation with n variables, then:

- the system of equations has exactly one solution if and only if A has exactly n pivot columns all of which lie to the left of the augment line;
- the system of equations has no solutions if and only if the right-most column of A is a pivot column;
- the system of equations has an unlimited number of solutions if and only if there are less than n pivot columns in A and the right-most column of A is not a pivot column.

A system of equations with at least one solution is called ***consistent***. A system of equations with no solutions is called ***inconsistent***.

Gauss-Jordan Elimination Method

The ***Gauss-Jordan elimination method*** follows the same row manipulation rules as Gaussian elimination. What distinguishes the two methods is that the Gaussian method stops once you've reduced the matrix to row echelon form where-as the Gauss-Jordanian method requires you to further reduce the matrix to ***reduced (row) echelon form***. That is, the simplified matrix must meet each of the following properties:

i. Every row that contains nothing but zeros occurs at the bottom of the matrix.
ii. The leading entry of any non-zero row appears to the right of the leading entry in the row directly above it.
iii. All entries directly above or below a pivot position are zero.
iv. Every leading entry is 1.

When manipulating a matrix into reduced row echelon form, there are essentially three tasks that need to be completed. In the list below, you always want to complete task A first. Tasks B and C can be done in either order.

A. Manipulate every entry below a leading entry to zero. This task should be worked left-to-right; i.e., first create the necessary zeros in the first column, then the second column, then the third column, etc. Remember that the leading entries need to move rightward as you move down the rows of the matrix. Occasionally you will need to swap rows to maintain this cascading effect. **Always use the row with the leading entry to create the zeros below the leading entry.**
B. Manipulate every entry above a leading entry to zero. This task should be worked right-to-left. **Always use the row with the leading entry to create the zeros above the leading entry.**
C. Multiply each of the rows by the necessary constants so that each leading entry is 1.

Definitions and Fundamental Facts

Suppose that A is the augmented representation of a ***consistent system*** with *n* variables that has ***an unlimited number of solutions***.

The ***free variables*** in the ***general solution set*** correspond to the non-pivot columns of A that lie to the left of the augment line. The other variables in the system are called ***basic variables***. The value of any given basic variable might be fixed or it might be dependent upon the value(s) assigned to one or more of the free variables.

Specific solutions to the system are determined by assigning specific values to the free variables and then determining the associated values of the basic variables.