Example 10.1

Determine the eigenvalues and eigenvectors of the matrix A where $A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$.

Example 10.2

Whence the characteristic equation?

Example 10.3

Determine the eigenvalues for $A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ -4 & 6 & 0 & 0 \\ -1 & 12 & 3 & 0 \\ 4 & 4 & 2 & 0 \end{bmatrix}$.

Example 10.4

Determine bases for the eigenspaces of B where $B = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$.

Example 10.5

Determine two distinct diagonalizations of A where $A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$.

Example10.6

Diagonalize B where $B = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}$; use the result to simplify B^n where n is a natural number. To get to the point more quickly, let's use our calculators to find the eigenvalues of M.

Example 10.7

What happens when we try to diagonalize M where $M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -5 & 4 \end{bmatrix}$? To get to the point more expeditiously, let's use our calculators to find the eigenvalues of M.

Example 10.8

Consider the recursive sequence where $a_1 = 1$, $a_2 = 1$, and $a_k = 2a_{k-1} + 3a_{k-2}$ for $k \geq 3$. Let's find a general term formula (non-recursive) for a_k starting at $k = 3$.

Example 10.9

Diagonalize T where $T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

Example 10.10

Explain ***geometrically*** why the rotation matrix $R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ cannot possibly have any real number eigenvalues for $0 < \theta < \pi$.

Definitions 10.1-10.3: Eigenvalues and Eigenvectors (of square matrices)

A non-zero vector $\vec{v}$ is called ***an eigenvector*** of the square matrix A if there exists a scalar, λ, with the property that $A\vec{v} = \lambda\vec{v}$. If such a vector and scalar exist, the scalar λ is called ***an eigenvalue*** of A.

The eigenvalues of A are the solutions to the equation $\det(A - \lambda I) = 0$; this equation is called ***the characteristic equation*** of A.

Definitions 10.4 and 10.5: Eigenspaces (of square matrices)

The set of all eigenvectors associated with the specific eigenvalue λ_i is called the ***λ_i-eigenspace of A***. The dimension of the λ_i-eigenspace is called the geometric multiplicity of λ_i.

Definition 10.6 and Theorem 10.1: Similar Matrices

The square matrices A and B are ***similar matrices*** if and only if there exists a matrix P with the property that $A = PBP^{-1}$ (or, similarly, $P^{-1}AP = B$). Similar matrices have the same characteristic equation.

NOTE: Not all matrices that share a characteristic equation are similar!

Theorem 10.2 and Definition 10.: 7Diagonalization of an $n \times n$ matrix *A*

If A has n linearly independent eigenvectors, then A is similar to a diagonal matrix, D. Furthermore, $D = P^{-1}AP$ where the columns of P are composed of n linearly independent eigenvectors of A and the main diagonal entry in the i^{th} column of D is the eigenvalue that corresponds to the eigenvector in the i^{th} column of P. The product PDP^{-1} is called a ***diagonalization*** of A.