1. Consider the subspace of $\mathbb{R}^3$ consisting of vectors of form $\begin{bmatrix} a \\ b \\ 2a+b \end{bmatrix}$; call the set H.

   a. True or false? The vectors in $\left\{ \begin{bmatrix} 3 \\ 4 \\ 10 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix} \right\}$ form a basis for H? Explain!

   b. True or false? The vectors in $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix} \right\}$ form a basis for H? Explain!

   c. True or false? The vectors in $\left\{ \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 9 \end{bmatrix} \right\}$ form a basis for H? Explain!

2. Consider $A = \begin{bmatrix} 1 & -2 & 2 & 1 \\ 0 & 3 & 1 & 3 \\ -2 & 4 & 1 & -2 \\ 5 & 0 & 0 & 15 \\ 0 & 4 & 0 & 4 \end{bmatrix}$. Let's call the columns of the matrix $\vec{C}_1, \vec{C}_2, \vec{C}_3, \vec{C}_4$.

   a. True or false? The column space of A is all of $\mathbb{R}^5$? Explain!

   b. True or false? Every vector from the column space of A can be written as a linear combination of the vectors $\vec{e}_1 - \vec{e}_5$ from $\mathbb{R}^5$? Explain!

   c. True or false? The vectors $\vec{e}_1 - \vec{e}_5$ from $\mathbb{R}^5$ form a basis for $\operatorname{col}(A)$? Explain!

   d. True or false? The vectors in $\{\vec{C}_1, \vec{C}_2, \vec{C}_3, \vec{C}_4\}$ form a basis for $\operatorname{col}(A)$? Explain!

   e. True or false? The vectors in $\{\vec{C}_1, \vec{C}_2, \vec{C}_3\}$ form a basis for $\operatorname{col}(A)$.? Explain!

   f. True or false? The vectors in $\{\vec{C}_1, \vec{C}_2, \vec{C}_4\}$ form a basis for $\operatorname{col}(A)$? Explain!

   g. True or false? The vectors in $\{\vec{C}_2, \vec{C}_3, \vec{C}_4\}$ form a basis for $\operatorname{col}(A)$? Explain!

   h. True or false? The null space of A is a subspace of $\mathbb{R}^5$? Explain!

   i. Consider the linear transformation $T(\vec{x}) = A\vec{x}$. True or false? The domain of T is $\mathbb{R}^5$? Explain!

   j. True or false? The vector $\vec{e}_5$ from $\mathbb{R}^5$ is in the codomain of T? Explain!

   k. True or false? The vector $\vec{e}_5$ from $\mathbb{R}^5$ is in the range of T? Explain!

1. An eigenvector for each of the following matrices is given. In each case, determine the eigenvalue associated with the given eigenvector. **No calculator usage allowed.**

   a. $\begin{bmatrix} 18 \\ 6 \\ 1 \end{bmatrix}$ is an eigenvector for $\begin{bmatrix} 0 & 4 & 3 \\ 1/2 & 0 & 0 \\ 0 & 1/4 & 0 \end{bmatrix}$

   b. $\begin{bmatrix} -5 \\ 1 \\ 7 \end{bmatrix}$ is an eigenvector for $\begin{bmatrix} 1 & -2 & 1 \\ -3 & -1 & -2 \\ -7 & 7 & -6 \end{bmatrix}$

2. Use the characteristic equation $\det(A - \lambda I) = 0$ to determine the eigenvalues for each of the given matrices. **No calculator usage allowed.**

   a. $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

   b. $A = \begin{bmatrix} 0 & 4 \\ -1 & 5 \end{bmatrix}$

3. The eigenvalues for each of the following matrices are given. In each case, use the equation $(A - \lambda I)\vec{x} = \vec{0}$ to help you determine bases for the associated eigenspaces. You may use your calculator to row reduce a matrix when row reduction is necessary.

   a. The eigenvalues for $A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$ are 3 and -2 .

   b. The eigenvalues for $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$ are 3, 1 and -1.

4. The eigenvalues and bases for the associated eigenspaces for the matrix $B = \begin{bmatrix} 7 & 1 & -2 \\ -3 & 3 & 6 \\ 2 & 2 & 2 \end{bmatrix}$ are 0 with basis $\left\{ \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} \right\}$ and 6 with basis $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$. Use this information to determine a diagonalization of B; i.e., determine matrices P and D such that $B = PDP^{-1}$ where D is a diagonal matrix. Verify your result by hand. You may use your calculator to determine P^{-1}.

5. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$. Find a formula for A^k . Then use your formula to determine A^{10} and verify the result with your calculator. Note : to begin you need to diagonalize A. **No calculator usage allowed.**