

MTH 261 Graded HW 9

Name

Key

This assignment is due at 6:00 PM on Monday, June 5

You may work on this assignment with your classmates or anybody else you please. You may get help from a tutor or even the instructor. What you may not do is simply copy somebody else's work – that completely obviates the purpose of the assignment. If you forget to complete the assignment before it is due, do not simply copy someone else's paper and turn that in ... that is not "working together," that is taking credit for somebody else's work. You should not be working on this in class right before it is due; you have a five days to get this done – it should be done well before ten minutes before it is due.

1. Consider the recursive sequence where $a_1 = 2$, $a_2 = 6$, and $a_k = 0.5a_{k-1} + 1.5a_{k-2}$ for $k \geq 3$.

Determine a general term formula (non-recursive) for a_k starting at $k=3$. Work this entire problem without your calculator and make sure that you show all relevant work as well as a clear conclusion.

$$\text{For } k \geq 3, \begin{bmatrix} a_k \\ a_{k-1} \end{bmatrix} = \begin{bmatrix} .5 & 1.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_{k-1} \\ a_{k-2} \end{bmatrix}$$

$$\text{Define } A = \begin{bmatrix} .5 & 1.5 \\ 1 & 0 \end{bmatrix}.$$

Finding the eigenvalues of $A \dots$

$$\begin{vmatrix} .5 - \lambda & 1.5 \\ 1 & -\lambda \end{vmatrix} = 0 \Rightarrow -\lambda(.5 - \lambda) - 1.5 = 0$$

$$\Rightarrow \lambda^2 - .5\lambda - 1.5 = 0$$

$$\Rightarrow (\lambda + 1)(\lambda - 1.5) = 0$$

$$\Rightarrow \lambda = -1 \text{ or } \lambda = 1.5$$

-1 - eigenspace

$$\begin{bmatrix} .5 & 1.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 \\ -x_2 \end{bmatrix}$$

$$\begin{bmatrix} 1.5 & 1.5 & | & 0 \\ 1 & 1 & | & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

Gen Sol: $\begin{cases} x_1 = -x_2 \\ x_2 \text{ is free} \end{cases}$ Basis: $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$

1.5 - eigenspace

$$\begin{bmatrix} .5 & 1.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.5x_1 \\ 1.5x_2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1.5 & | & 0 \\ 1 & -1.5 & | & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1.5 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

Gen Sol: $\begin{cases} x_1 = 1.5x_2 \\ x_2 \text{ is free} \end{cases}$

Basis

$$\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$$

$$\therefore A = PDP^{-1} \text{ where } P = \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix}, D = \begin{bmatrix} -1 & 0 \\ 0 & 1.5 \end{bmatrix}, P^{-1} = \frac{1}{-5} \begin{bmatrix} 2 & -3 \\ -1 & -1 \end{bmatrix} \\ = \begin{bmatrix} -.4 & .6 \\ .2 & .2 \end{bmatrix}$$

This gives us (for $k \geq 3$):

$$\begin{bmatrix} a_k \\ a_{k-1} \end{bmatrix} = A^{k-2} \begin{bmatrix} a_2 \\ a_1 \end{bmatrix} \\ = \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} (-1)^{k-2} & 0 \\ 0 & 1.5^{k-2} \end{bmatrix} \begin{bmatrix} -.4 & .6 \\ .2 & .2 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \end{bmatrix} \\ = \begin{bmatrix} (-1)^{k-1} & 3(1.5)^{k-2} \\ (-1)^k & 2(1.5)^{k-2} \end{bmatrix} \begin{bmatrix} -1.2 \\ 1.6 \end{bmatrix} \\ = \begin{bmatrix} -1.2(-1)^{k-1} + 4.8(1.5)^{k-2} \\ -1.2(-1)^{k-2} + 3.2(1.5)^{k-2} \end{bmatrix}$$

$$\therefore \text{ For } k \geq 3, a_k = -1.2(-1)^{k-1} + 4.8(1.5)^{k-2}$$

