

MTH 261 Graded HW 8

Name

Key

This assignment is due at 6:00 PM on Monday, June 5

You may work on this assignment with your classmates or anybody else you please. You may get help from a tutor or even the instructor. What you may not do is simply copy somebody else's work – that completely obviates the purpose of the assignment. If you forget to complete the assignment before it is due, do not simply copy someone else's paper and turn that in ... that is not "working together," that is taking credit for somebody else's work. You should not be working on this in class right before it is due; you have a five days to get this done – it should be done well before ten minutes before it is due.

1. Suppose that A is a 3×3 matrix and that a basis for the null space of A is $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} \right\}$.

Determine a basis for the row space of A . Make sure that your thought process and your conclusion are both clear.

From the two basis vectors for the null space of A we can infer that the general solution to $A\vec{x} = \vec{0}$ is $\begin{cases} x_1 = -2x_2 + 4x_3 \\ x_2 \text{ is free} \\ x_3 \text{ is free} \end{cases}$

from which we can infer that

$$\text{rref}(A) = \begin{bmatrix} 1 & 2 & -4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

\therefore A basis for the row space of A is: $\{ [1, 2, -4] \}$

2. Jesse will only wear red, green, or blue shirts. Suppose that a Markov chain is established for Jesse's daily wardrobe choices and suppose that the state vectors are apportioned as follows.

$$v_k = \begin{bmatrix} \text{probability that Jesse wears a red shirt on "day k"} \\ \text{probability that Jesse wears a green shirt on "day k"} \\ \text{probability that Jesse wears a blue shirt on "day k"} \end{bmatrix}$$

The actual questions appear on page 2.

Suppose further that the transition matrix for the Markov chain is $A = \begin{bmatrix} .5 & .75 & .75 \\ .25 & 0 & .25 \\ .25 & .25 & 0 \end{bmatrix}$.

- a. What is the contextual significance of the entry $a_{12} = 0.75$?

On any day that Jesse wears a green shirt, there is a 75% probability that he will wear a red shirt the next day.

- b. Determine the stochastic steady state vector for the Markov chain. Show all relevant work and make sure that your conclusion is clear.

Steady state vectors for A satisfy $A\vec{x} = \vec{x}$.

$$\begin{bmatrix} -.5 & .75 & .75 & | & 0 \\ .25 & -1 & .25 & | & 0 \\ .25 & .25 & -1 & | & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$\text{General Solution: } \begin{cases} x_1 = 3x_3 \\ x_2 = x_3 \\ x_3 \text{ is free} \end{cases}$$

\therefore A steady state vector is $\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$ and the Stochastic Steady state vector is $\begin{bmatrix} 3/5 \\ 1/5 \\ 1/5 \end{bmatrix}$

- c. What is the contextual significance of the bottom entry of the stochastic steady state vector you found in part (b) of the question?

Jesse wears a blue shirt 20% of the time.