MTH 261 Graded HW 6 **Name** Key

This assignment is due at 6:00 PM on Monday, May 15

You should not be working on this in class right before it is due; you have 5 days to get this done – it should be done well before ten minutes before it is due.

1. Demonstrate, both implicitly and explicitly, that the linear transformation T, given below, is neither one-to-one nor onto $\mathbb{R}^3$. (See Example 7.6.)

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 - x_3 \\ x_1 - x_2 - x_3 \\ x_1 + 7x_2 - x_3 \end{bmatrix}$$

$T(\vec{x}) = A\vec{x}$ where $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & 7 & -1 \end{bmatrix}$

$\det(A) = 0$ (calculator), $\therefore$ T is neither one-to-one or onto $\mathbb{R}^3$ (Theorem 8, properties f, i, and m).

Solving $A\vec{x} = \vec{0}$ we have $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & -1 & -1 & 0 \\ 1 & 7 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

The general solution is $\begin{cases} x_1 = x_3 \\ x_2 = 0 \\ x_3 \text{ is free.} \end{cases}$

$\therefore$ $T(\vec{0}) = \vec{0}$ and $T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & 7 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \vec{0}$

which explicitly shows that T is not one-to-one

On to onto...

$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 2 \\ 1 & 7 & -1 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$. The contradiction in the third row of the reduced matrix establishes that $T(\vec{x}) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ has no solutions, thus explicitly showing that T is not onto $\mathbb{R}^3$.

2. Determine and state a basis for the row space of the matrix A (given below) – show the necessary work (you may use your calculator). Then express each row of A as a linear combination of the stated basis vectors.

$$A = \begin{bmatrix} 2 & -4 & -3 & -3 & 9 \\ -2 & 4 & 1 & 5 & -7 \\ 1 & -2 & 2 & -5 & 1 \end{bmatrix}$$

$$A \sim \begin{bmatrix} 1 & -2 & 0 & -3 & 3 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A basis for row(A) is $\{[1,-2,0,-3,3], [0,0,1,-1,-1]\}$

Expressing the rows of A in terms of this basis...

$[2,-4,-3,-3,9] = 2[1,-2,0,-3,3] + (-3)[0,0,1,-1,-1]$

$[-2,4,1,5,-7] = -2[1,-2,0,-3,3] + [0,0,1,-1,-1]$

$[1,-2,2,-5,1] = [1,-2,0,-3,3] + 2[0,0,1,-1,-1]$

3. Let $A = \begin{bmatrix} 2 & -4 & -3 & 17 & 5 & 4 \\ 3 & -6 & -2 & 18 & 5 & 2 \\ -1 & 2 & 3 & -13 & -4 & -1 \\ 4 & -8 & 1 & 13 & 3 & 1 \end{bmatrix}$. Note that $A \sim \begin{bmatrix} 1 & -2 & 0 & 4 & 1 & 0 \\ 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Fill-in each of the following blanks.

The column space of M is a ___3___-dimensional subspace of $\mathbb{R}$_4_.

The row space of M is a ___3___-dimensional subspace of $\mathbb{R}$_6_.

The null space of M is a ___3___-dimensional subspace of $\mathbb{R}$_6_.

The column space of M^{T} is a ___3___-dimensional subspace of $\mathbb{R}$_6_.

The row space of M^{T} is a ___3___-dimensional subspace of $\mathbb{R}$_4_.

The null space of M^{T} is a ___1___-dimensional subspace of $\mathbb{R}$_4_.