

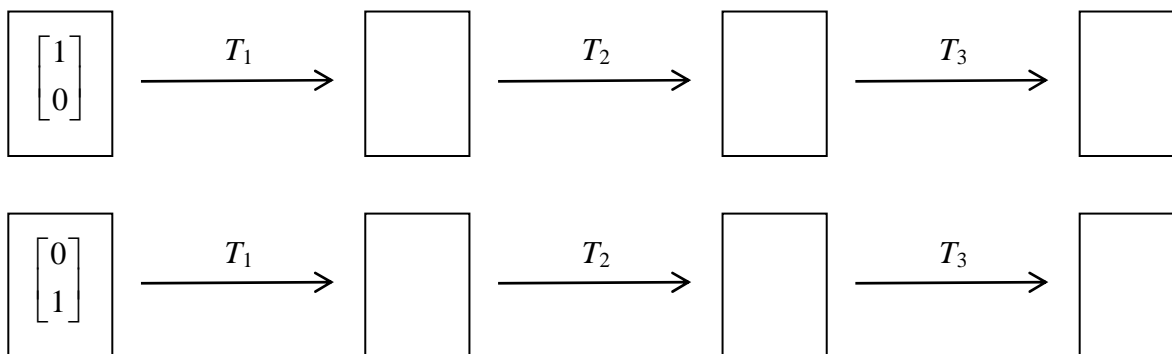
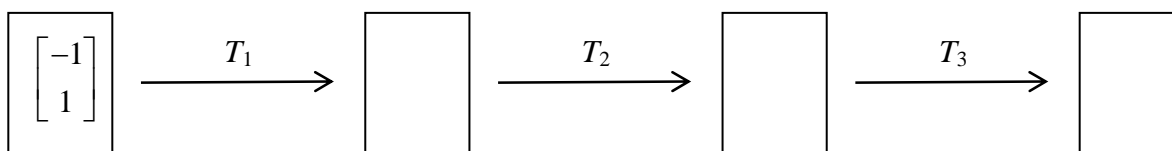
## MTH 261 Graded HW 5

Name \_\_\_\_\_

**This assignment is due at 6:00 PM on Wednesday, May 10****You should not be working on this in class right before it is due; you have a week to get this done – it should be done well before ten minutes before it is due.**

1. Consider the linear transformations described below.

- $T_1 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  by reflecting vectors across the line  $y = x$ .
- $T_2 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  by rotating vectors  $45^\circ$  in the clockwise direction.
- $T_3 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  by reflecting vectors across the line  $y = -x$ .
- $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  such that  $T(\vec{x}) = T_3(T_2(T_1(\vec{x})))$ .

a. Track the two standard unit vectors from  $\mathbb{R}^2$  through the three step  $T$  process. Then write down the implied linear transformation matrix,  $M$ , where  $T(\vec{x}) = M \vec{x}$ . $M =$ b. Track the vector  $\vec{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$  through the three step process  $T$  process and then verify that  $M \vec{x} = T(\vec{x})$ . $M \vec{x} =$

1. (Continued)

c. Write down the linear transformation matrices,  $M_1$ ,  $M_2$ , and  $M_3$  such that  $T_1(\vec{x}) = M_1 \vec{x}$ ,  $T_2(\vec{x}) = M_2 \vec{x}$ , and  $T_3(\vec{x}) = M_3 \vec{x}$ .

d. Multiply the matrices  $M_1$ ,  $M_2$ , and  $M_3$  in the proper order so that the result is the matrix  $M$ . Show the work.

2. Suppose that  $T_1$  rotates vectors from  $\mathbb{R}^2$  counterclockwise by  $120^\circ$  whereas  $T_2$  rotates vectors from  $\mathbb{R}^2$  clockwise by  $120^\circ$ . Furthermore, suppose that  $T$  is the linear transformation that applies  $T_1$  followed by  $T_2$  (i.e.,  $T(\vec{x}) = T_2(T_1(\vec{x}))$ ).
- What is the net effect of applying  $T_1$  followed by  $T_2$ . That is, what is the relative position of  $\vec{x}$  and  $T(\vec{x})$ ?
  - Given your answer to part (a), what is the linear transformation matrix for  $T$ ?
  - Suppose that  $A_1$  and  $A_2$  are the linear transformation matrices with the properties that  $T_1(\vec{x}) = A_1 \vec{x}$  and  $T_2(\vec{x}) = A_2 \vec{x}$ . Given your answer to (b) and the fact that  $T$  can be effected using  $T(\vec{x}) = A_2(A_1 \vec{x})$ , and the fact that  $A_2(A_1 \vec{x}) = (A_2 A_1) \vec{x}$ , what must be the relationship between  $A_1$  and  $A_2$ ?
  - Write down the matrices  $A_1$  and  $A_2$  such that  $T_1(\vec{x}) = A_1 \vec{x}$  and  $T_2(\vec{x}) = A_2 \vec{x}$ ; replace the sine and cosine expressions with their actual (exact) values. Then confirm the relationship you stated in part (c).