MTH 261 Graded HW 5 **Name** ______________________________

This assignment is due at 6:00 PM on Wednesday, May 10

<u>You should not be working on this in class right before it is due; you have a week to get this done – it should be done well before ten minutes before it is due</u>.

1. Consider the linear transformations described below.
   - $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ by reflecting vectors across the line $y = x$.
   - $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ by rotating vectors 45° in the clockwise direction.
   - $T_3 : \mathbb{R}^2 \to \mathbb{R}^2$ by reflecting vectors across the line $y = -x$.
   - $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(\vec{x}) = T_3\left(T_2\left(T_1(\vec{x})\right)\right)$.

a. Track the two standard unit vectors from $\mathbb{R}^2$ through the three step T process. Then write down the implied linear transformation matrix, M, where $T(\vec{x}) = M\,\vec{x}$.

$$\boxed{\begin{bmatrix}1\\0\end{bmatrix}} \xrightarrow{T_1} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}} \xrightarrow{T_2} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}} \xrightarrow{T_3} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}}$$

$$\boxed{\begin{bmatrix}0\\1\end{bmatrix}} \xrightarrow{T_1} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}} \xrightarrow{T_2} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}} \xrightarrow{T_3} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}}$$

$M =$

b. Track the vector $\vec{x} = \begin{bmatrix}-1\\1\end{bmatrix}$ through the three step process T process and then verify that $M\,\vec{x} = T(\vec{x})$.

$$\boxed{\begin{bmatrix}-1\\1\end{bmatrix}} \xrightarrow{T_1} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}} \xrightarrow{T_2} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}} \xrightarrow{T_3} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}}$$

$M\,\vec{x} =$

1. (Continued)

c. Write down the linear transformation matrices, M_1, M_2, and M_3 such that $T_1(\vec{x}) = M_1\,\vec{x}$, $T_2(\vec{x}) = M_2\,\vec{x}$, and $T_3(\vec{x}) = M_3\,\vec{x}$.

d. Multiply the matrices M_1, M_2, and M_3 in the proper order so that the result is the matrix M. Show the work.

2. Suppose that T_1 rotates vectors from $\mathbb{R}^2$ counterclockwise by 120^{o} whereas T_2 rotates vectors from $\mathbb{R}^2$ clockwise by 120^{o}. Furthermore, suppose that T is the linear transformation that applies T_1 followed by T_2 (i.e., $T(\vec{x}) = T_2(T_1(\vec{x}))$).

   a. What is the net effect of applying T_1 followed by T_2. That is, what is the relative position of $\vec{x}$ and $T(\vec{x})$?

   b. Given your answer to part (a), what is the linear transformation matrix for T?

   c. Suppose that A_1 and A_2 are the linear transformation matrices with the properties that $T_1(\vec{x}) = A_1\vec{x}$ and $T_2(\vec{x}) = A_2\vec{x}$. Given your answer to (b) and the fact that T can be effected using $T(\vec{x}) = A_2(A_1\vec{x})$, and the fact that $A_2(A_1\vec{x}) = (A_2 A_1)\vec{x}$, what must be the relationship be between A_1 and A_2 ?

   d. Write down the matrices A_1 and A_2 such that $T_1(\vec{x}) = A_1\vec{x}$ and $T_2(\vec{x}) = A_2\vec{x}$; replace the sine and cosine expressions with their actual (exact) values. Then confirm the relationship you stated in part (c).