MTH 261 Graded HW 4 **Name** Key

This assignment is due at 6:00 PM on Wednesday, May 10

<u>You should not be working on this in class right before it is due; you have a week to get this done – it should be done well before ten minutes before it is due</u>.

1. Let $A = \begin{bmatrix} 3 & 2 & -4 \\ -1 & 1 & 6 \\ 2 & 5 & 3 \end{bmatrix}$. Use the determinant and adjoint of A to calculate A^{-1}. To earn full credit your work must be shown on this paper in a manner consistent with what was illustrated in class. Make sure that you show work consistent with that shown in the lecture notes Example 5.10.

$$C_{11} = (-1)^{1+1}\begin{vmatrix} 1 & 6 \\ 5 & 3 \end{vmatrix} = -27 \qquad C_{12} = (-1)^{1+2}\begin{vmatrix} -1 & 6 \\ 2 & 3 \end{vmatrix} = 15 \qquad C_{13} = (-1)^{1+3}\begin{vmatrix} -1 & 1 \\ 2 & 5 \end{vmatrix} = -7$$

$$C_{21} = (-1)^{2+1}\begin{vmatrix} 2 & -4 \\ 5 & 3 \end{vmatrix} = -26 \qquad C_{22} = (-1)^{2+2}\begin{vmatrix} 3 & -4 \\ 2 & 3 \end{vmatrix} = 17 \qquad C_{23} = (-1)^{2+3}\begin{vmatrix} 3 & 2 \\ 2 & 5 \end{vmatrix} = -11$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} 2 & -4 \\ 1 & 6 \end{vmatrix} = 16 \qquad C_{32} = (-1)^{3+2}\begin{vmatrix} 3 & -4 \\ -1 & 6 \end{vmatrix} = -14 \qquad C_{33} = (-1)^{3+3}\begin{vmatrix} 3 & 2 \\ -1 & 1 \end{vmatrix} = 5$$

$$\det(A) = a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33}$$
$$= (-4)(-7) + (6)(-11) + (3)(5)$$
$$= -23$$

$$A^{-1} = \frac{1}{\det(A)}\,\mathrm{adj}(A) = \frac{1}{-23}\begin{bmatrix} -27 & 15 & -7 \\ -26 & 17 & -11 \\ 16 & -14 & 5 \end{bmatrix}^T = \frac{1}{23}\begin{bmatrix} 27 & 26 & -16 \\ -15 & -17 & 14 \\ 7 & 11 & -5 \end{bmatrix}$$

2. Determine the linear transformation matrix for T given that $T\left(\begin{bmatrix}3\\7\end{bmatrix}\right)=\begin{bmatrix}7\\32\end{bmatrix}$ and $T\left(\begin{bmatrix}1\\3\end{bmatrix}\right)=\begin{bmatrix}1\\14\end{bmatrix}$.

Make sure that you show work consistent with that shown in the lecture notes Example 6.9.

Let's begin by solving:

$$x_1\begin{bmatrix}3\\7\end{bmatrix}+x_2\begin{bmatrix}1\\3\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix} \text{ and } y_1\begin{bmatrix}3\\7\end{bmatrix}+y_2\begin{bmatrix}1\\3\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}$$

$$\left[\begin{array}{cc|cc}3 & 1 & 1 & 0\\ 7 & 3 & 0 & 1\end{array}\right] \sim \left[\begin{array}{cc|cc}1 & 0 & 3/2 & -1/2\\ 0 & 1 & -7/2 & 3/2\end{array}\right]$$

$$\therefore \begin{bmatrix}1\\0\end{bmatrix}=(3/2)\begin{bmatrix}3\\7\end{bmatrix}+(-7/2)\begin{bmatrix}1\\3\end{bmatrix} \text{ and } \begin{bmatrix}0\\1\end{bmatrix}=(-1/2)\begin{bmatrix}3\\7\end{bmatrix}+(3/2)\begin{bmatrix}1\\3\end{bmatrix}$$

$$\therefore \quad T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)=\frac{3}{2}T\left(\begin{bmatrix}3\\7\end{bmatrix}\right)-\frac{7}{2}T\left(\begin{bmatrix}1\\3\end{bmatrix}\right)$$

$$=\frac{3}{2}\begin{bmatrix}7\\32\end{bmatrix}-\frac{7}{2}\begin{bmatrix}1\\14\end{bmatrix}$$

$$=\begin{bmatrix}7\\-1\end{bmatrix}$$

and

$$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)=-\frac{1}{2}T\left(\begin{bmatrix}3\\7\end{bmatrix}\right)+\frac{3}{2}T\left(\begin{bmatrix}1\\3\end{bmatrix}\right)$$

$$=-\frac{1}{2}\begin{bmatrix}7\\32\end{bmatrix}+\frac{3}{2}\begin{bmatrix}1\\14\end{bmatrix}$$

$$=\begin{bmatrix}-2\\5\end{bmatrix}$$

$$\therefore \quad T(\vec{x})=A\vec{x} \quad \text{where} \quad A=\begin{bmatrix}7 & -2\\ -1 & 5\end{bmatrix}$$

Check: $\begin{bmatrix}7 & -2\\ -1 & 5\end{bmatrix}\begin{bmatrix}3\\7\end{bmatrix}=\begin{bmatrix}7\\32\end{bmatrix}$ ✓ and $\begin{bmatrix}7 & -2\\ -1 & 5\end{bmatrix}\begin{bmatrix}1\\3\end{bmatrix}=\begin{bmatrix}1\\14\end{bmatrix}$