

MTH 261 Graded HW 4

Name _____

This assignment is due at 6:00 PM on Wednesday, May 10

You should not be working on this in class right before it is due; you have a week to get this done – it should be done well before ten minutes before it is due.

1. Let $A = \begin{bmatrix} 3 & 2 & -4 \\ -1 & 1 & 6 \\ 2 & 5 & 3 \end{bmatrix}$. Use the determinant and adjoint of A to calculate A^{-1} . To earn full credit

your work must be shown on this paper in a manner consistent with what was illustrated in class.
Make sure that you show work consistent with that shown in the lecture notes Example 5.10.

2. Determine the linear transformation matrix for T given that $T\left(\begin{bmatrix} 3 \\ 7 \end{bmatrix}\right) = \begin{bmatrix} 7 \\ 32 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 14 \end{bmatrix}$.
Make sure that you show work consistent with that shown in the lecture notes Example 6.9.

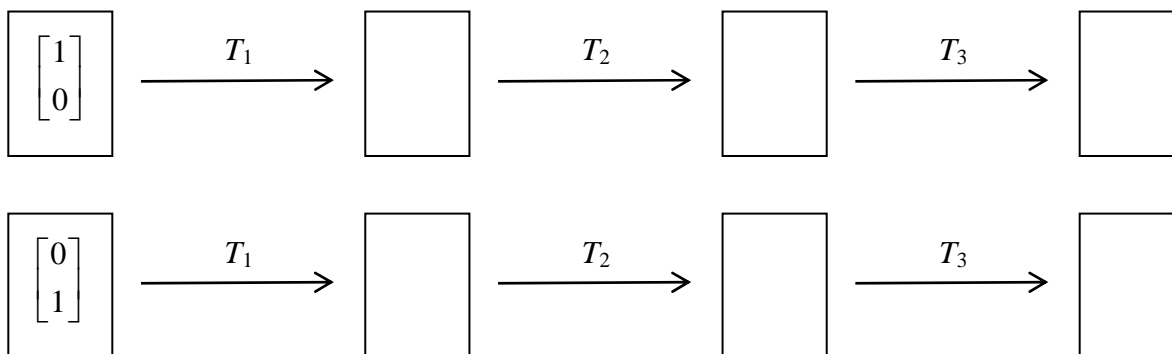
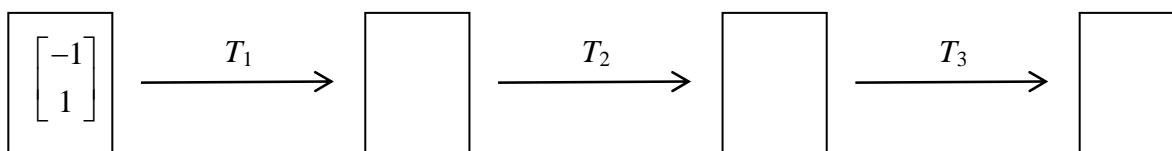
MTH 261 Graded HW 5

Name _____

This assignment is due at 6:00 PM on Wednesday, May 10**You should not be working on this in class right before it is due; you have a week to get this done – it should be done well before ten minutes before it is due.**

1. Consider the linear transformations described below.

- $T_1 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by reflecting vectors across the line $y = x$.
- $T_2 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by rotating vectors 45° in the clockwise direction.
- $T_3 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by reflecting vectors across the line $y = -x$.
- $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ such that $T(\vec{x}) = T_3(T_2(T_1(\vec{x})))$.

a. Track the two standard unit vectors from \mathbb{R}^2 through the three step T process. Then write down the implied linear transformation matrix, M , where $T(\vec{x}) = M\vec{x}$. $M =$ b. Track the vector $\vec{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ through the three step process T process and then verify that $M\vec{x} = T(\vec{x})$. $M\vec{x} =$

1. (Continued)

c. Write down the linear transformation matrices, M_1 , M_2 , and M_3 such that $T_1(\vec{x}) = M_1 \vec{x}$, $T_2(\vec{x}) = M_2 \vec{x}$, and $T_3(\vec{x}) = M_3 \vec{x}$.

d. Multiply the matrices M_1 , M_2 , and M_3 in the proper order so that the result is the matrix M . Show the work.

2. Suppose that T_1 rotates vectors from \mathbb{R}^2 counterclockwise by 120° whereas T_2 rotates vectors from \mathbb{R}^2 clockwise by 120° . Furthermore, suppose that T is the linear transformation that applies T_1 followed by T_2 (i.e., $T(\vec{x}) = T_2(T_1(\vec{x}))$).
- What is the net effect of applying T_1 followed by T_2 . That is, what is the relative position of \vec{x} and $T(\vec{x})$?
 - Given your answer to part (a), what is the linear transformation matrix for T ?
 - Suppose that A_1 and A_2 are the linear transformation matrices with the properties that $T_1(\vec{x}) = A_1 \vec{x}$ and $T_2(\vec{x}) = A_2 \vec{x}$. Given your answer to (b) and the fact that T can be effected using $T(\vec{x}) = A_2(A_1 \vec{x})$, and the fact that $A_2(A_1 \vec{x}) = (A_2 A_1) \vec{x}$, what must be the relationship between A_1 and A_2 ?
 - Write down the matrices A_1 and A_2 such that $T_1(\vec{x}) = A_1 \vec{x}$ and $T_2(\vec{x}) = A_2 \vec{x}$; replace the sine and cosine expressions with their actual (exact) values. Then confirm the relationship you stated in part (c).