**MTH 261 Graded HW 4** **Name** ______________________________

**This assignment is due at 6:00 PM on Wednesday, May 10**

**You should not be working on this in class right before it is due; you have a week to get this done – it should be done well before ten minutes before it is due**.

1. Let $A=\begin{bmatrix} 3 & 2 & -4 \\ -1 & 1 & 6 \\ 2 & 5 & 3 \end{bmatrix}$. Use the determinant and adjoint of $A$ to calculate $A^{-1}$. To earn full credit your work must be shown on this paper in a manner consistent with what was illustrated in class. Make sure that you show work consistent with that shown in the lecture notes Example 5.10.

**2.** Determine the linear transformation matrix for $T$ given that $T\left(\begin{bmatrix}3\\7\end{bmatrix}\right)=\begin{bmatrix}7\\32\end{bmatrix}$ and $T\left(\begin{bmatrix}1\\3\end{bmatrix}\right)=\begin{bmatrix}1\\14\end{bmatrix}$.

Make sure that you show work consistent with that shown in the lecture notes Example 6.9.

**MTH 261 Graded HW 5** **Name** ______________________________________

**This assignment is due at 6:00 PM on Wednesday, May 10**

**<u>You should not be working on this in class right before it is due; you have a week to get this done – it should be done well before ten minutes before it is due</u>**.

1. Consider the linear transformations described below.
   - $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ by reflecting vectors across the line $y = x$.
   - $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ by rotating vectors 45$^{\circ}$ in the clockwise direction.
   - $T_3 : \mathbb{R}^2 \to \mathbb{R}^2$ by reflecting vectors across the line $y = -x$.
   - $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(\vec{x}) = T_3\left(T_2\left(T_1(\vec{x})\right)\right)$.

a. Track the two standard unit vectors from $\mathbb{R}^2$ through the three step $T$ process. Then write down the implied linear transformation matrix, $M$, where $T(\vec{x}) = M\,\vec{x}$.

$$\boxed{\begin{bmatrix}1\\0\end{bmatrix}} \xrightarrow{T_1} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}} \xrightarrow{T_2} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}} \xrightarrow{T_3} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}}$$

$$\boxed{\begin{bmatrix}0\\1\end{bmatrix}} \xrightarrow{T_1} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}} \xrightarrow{T_2} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}} \xrightarrow{T_3} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}}$$

$M =$

b. Track the vector $\vec{x} = \begin{bmatrix}-1\\1\end{bmatrix}$ through the three step process $T$ process and then verify that $M\,\vec{x} = T(\vec{x})$.

$$\boxed{\begin{bmatrix}-1\\1\end{bmatrix}} \xrightarrow{T_1} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}} \xrightarrow{T_2} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}} \xrightarrow{T_3} \boxed{\phantom{\begin{bmatrix}0\\0\end{bmatrix}}}$$

$M\,\vec{x} =$

1. (Continued)

c. Write down the linear transformation matrices, $M_1$, $M_2$ , and $M_3$ such that $T_1(\vec{x}) = M_1\vec{x}$, $T_2(\vec{x}) = M_2\vec{x}$, and $T_3(\vec{x}) = M_3\vec{x}$.

d. Multiply the matrices $M_1$, $M_2$ , and $M_3$ in the proper order so that the result is the matrix $M$. Show the work.

2. Suppose that $T_1$ rotates vectors from $\mathbb{R}^2$ counterclockwise by 120° whereas $T_2$ rotates vectors from $\mathbb{R}^2$ clockwise by 120°. Furthermore, suppose that $T$ is the linear transformation that applies $T_1$ followed by $T_2$ (i.e., $T(\vec{x}) = T_2(T_1(\vec{x}))$).

   a. What is the net effect of applying $T_1$ followed by $T_2$. That is, what is the relative position of $\vec{x}$ and $T(\vec{x})$ ?

   b. Given your answer to part (a), what is the linear transformation matrix for $T$?

   c. Suppose that $A_1$ and $A_2$ are the linear transformation matrices with the properties that $T_1(\vec{x}) = A_1\,\vec{x}$ and $T_2(\vec{x}) = A_2\,\vec{x}$. Given your answer to (b) and the fact that $T$ can be effected using $T(\vec{x}) = A_2(A_1\,\vec{x})$, and the fact that $A_2(A_1\,\vec{x}) = (A_2\,A_1)\vec{x}$, what must be the relationship be between $A_1$ and $A_2$ ?

   d. Write down the matrices $A_1$ and $A_2$ such that $T_1(\vec{x}) = A_1\,\vec{x}$ and $T_2(\vec{x}) = A_2\,\vec{x}$; replace the sine and cosine expressions with their actual (exact) values. Then confirm the relationship you stated in part (c).