**MTH 261 Graded HW 3** **Name** Key

**This assignment is due at 6:00 PM on Monday, April 24**

**You may work on this assignment with your classmates or anybody else you please. You may get help from a tutor or even the instructor. What you may not do is simply copy somebody else's work – that completely obviates the purpose of the assignment. If you forget to complete the assignment before it is due, do not simply copy someone else's paper and turn that in … that is not "working together," that is taking credit for somebody else's work. You should not be working on this in class right before it is due; you have a week to get this done – it should be done well before ten minutes before it is due.**

**Problem 1**

Let $A = \begin{bmatrix} 2 & 1 & 8 \\ -3 & 2 & -5 \\ 7 & -3 & 15 \end{bmatrix}$. **Explicitly demonstrate** that the column vectors of $A$ are linearly dependent.

Show all of the relevant work that went into the creation of your solution. You may use your calculator for row reduction on this problem. See notes example 3.6 for an example of an explicit demonstration of linear dependence.

The column vectors of A are linearly independent if and only if the vector equation below has non trivial solutions.

$$x_1 \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 8 \\ -5 \\ 15 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving: $\left[\begin{array}{ccc|c} 2 & 1 & 8 & 0 \\ -3 & 2 & -5 & 0 \\ 7 & -3 & 15 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

The general solution is $\begin{cases} x_1 = -3x_3 \\ x_2 = -2x_3 \\ x_3 \text{ is free.} \end{cases}$

So, for example, $(-3)\begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix} + (-2)\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} + (1)\begin{bmatrix} 8 \\ -5 \\ 15 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

QED

**Problem 2**

Use the row reduction algorithm (notes example 4.12) to find $A^{-1}$ where $A=\begin{bmatrix} 1 & -2 & 1 \\ 3 & -3 & 1 \\ -3 & 5 & -2 \end{bmatrix}$. Show all work in a manner consistent with that illustrated in class.

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 3 & -3 & 1 & 0 & 1 & 0 \\ -3 & 5 & -2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} -3R_1+R_2 \to R_2 \\ 3R_1+R_3 \to R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 3 & -2 & -3 & 1 & 0 \\ 0 & -1 & 1 & 3 & 0 & 1 \end{array}\right]$$

$$R_2 \leftrightarrow R_3 \quad \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 3 & 0 & 1 \\ 0 & 3 & -2 & -3 & 1 & 0 \end{array}\right]$$

$$3R_2+R_3 \to R_3 \quad \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 3 & 0 & 1 \\ 0 & 0 & 1 & 6 & 1 & 3 \end{array}\right]$$

$$\begin{array}{l} -R_3+R_1 \to R_1 \\ -R_3+R_2 \to R_2 \end{array} \quad \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & -5 & -1 & -3 \\ 0 & -1 & 0 & -3 & -1 & -2 \\ 0 & 0 & 1 & 6 & 1 & 3 \end{array}\right]$$

$$-R_2 \to R_2 \quad \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & -5 & -1 & -3 \\ 0 & 1 & 0 & 3 & 1 & 2 \\ 0 & 0 & 1 & 6 & 1 & 3 \end{array}\right]$$

$$2R_2+R_1 \to R_1 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 3 & 1 & 2 \\ 0 & 0 & 0 & 6 & 1 & 3 \end{array}\right]$$



$$\therefore A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 6 & 1 & 3 \end{bmatrix}$$