

## MTH 261 Graded HW 2

Name

Key

This assignment is due at 6:00 PM on Wednesday, April 19

You may work on this assignment with your classmates or anybody else you please. You may get help from a tutor or even the instructor. What you may not do is simply copy somebody else's work – that completely obviates the purpose of the assignment. If you forget to complete the assignment before it is due, do not simply copy someone else's paper and turn that in ... that is not "working together," that is taking credit for somebody else's work. You should not be working on this in class right before it is due; you have a week to get this done – it should be done well before ten minutes before it is due.

1. Determine the value of  $k$  so that the vector  $\begin{bmatrix} -2 \\ -12 \\ k \end{bmatrix}$  is in the span of  $\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \right\}$ . Show the relevant work and make sure that your reasoning and your conclusion are clear.

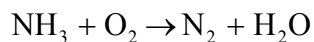
$\begin{bmatrix} -2 \\ -12 \\ k \end{bmatrix}$  is in the span of  $\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \right\}$  if and only if the equation  $x_1 \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ -12 \\ k \end{bmatrix}$  has at least one solution.

$$\begin{bmatrix} 1 & 1 & 1 & -2 \\ 1 & 3 & 1 & -12 \\ -2 & 2 & 1 & k \end{bmatrix} \xrightarrow{\substack{-R_1 + R_2 \rightarrow R_2 \\ 2R_1 + R_3 \rightarrow R_3}} \begin{bmatrix} 1 & 1 & 1 & -2 \\ 0 & 2 & 0 & -10 \\ 0 & 4 & 3 & k-4 \end{bmatrix} \xrightarrow{-2R_2 + R_3 \rightarrow R_3} \begin{bmatrix} 1 & 1 & 1 & -2 \\ 0 & 2 & 0 & -10 \\ 0 & 0 & 3 & k+16 \end{bmatrix}$$

From the bottom row of the last matrix, we can see that the stated vector equation will lead to a contradiction unless  $k = -16$ .

$\therefore \begin{bmatrix} -2 \\ -12 \\ k \end{bmatrix}$  is in the span of  $\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \right\}$  if and only if  $k = -16$ .

2. Using the techniques illustrated in Example 2.7 of your notes, balance the following chemical equation.



Let's define the molecules as column vectors thus:

$$\begin{bmatrix} \text{number of nitrogen atoms in one molecule} \\ \text{number of hydrogen atoms in one molecule} \\ \text{number of oxygen atoms in one molecule} \end{bmatrix}$$

The chemical equation is balanced by solutions to:

$$x_1 \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

This gives us the system of equations:

$$\begin{cases} x_1 - 2x_3 = 0 \\ 3x_1 - 2x_4 = 0 \\ 2x_2 - x_4 = 0 \end{cases}$$

$$\left[ \begin{array}{cccc|ccc} 1 & 0 & -2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & -2 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 & 1 \end{array} \right] \sim \left[ \begin{array}{cccc|ccc} 1 & 0 & 0 & -2/3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1/3 & 0 & 0 & 1 \end{array} \right]$$

So the general solution to our balance equation is:

$$\begin{cases} x_1 = \frac{2}{3}x_4 \\ x_2 = \frac{1}{2}x_4 \\ x_3 = \frac{1}{3}x_4 \\ x_4 \text{ is free} \end{cases}$$

Since we can't have fractional parts of molecules, we let  $x_4 = 6$ .

$\therefore$  A balanced equation is:

