MTH 261 Graded HW 2 **Name** Key

This assignment is due at 6:00 PM on Wednesday, April 19

You may work on this assignment with your classmates or anybody else you please. You may get help from a tutor or even the instructor. What you may not do is simply copy somebody else's work – that completely obviates the purpose of the assignment. If you forget to complete the assignment before it is due, do not simply copy someone else's paper and turn that in … that is not "working together," that is taking credit for somebody else's work. <u>You should not be working on this in class right before it is due; you have a week to get this done – it should be done well before ten minutes before it is due</u>.

1. Determine the value of k so that the vector $\begin{bmatrix} -2 \\ -12 \\ k \end{bmatrix}$ is in the span of $\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \right\}$. Show the relevant work and make sure that your reasoning and your conclusion are clear.

$\begin{bmatrix} -2 \\ -12 \\ k \end{bmatrix}$ is in the Span of $\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \right\}$ if and only if the equation $x_1 \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ -12 \\ k \end{bmatrix}$ has at least on solution.

$$\left[\begin{array}{cc|c} 1 & 1 & -2 \\ 1 & 3 & -12 \\ -2 & 2 & k \end{array}\right] \begin{array}{c} -R_1 + R_2 \to R_2 \\ 2R_1 + R_3 \to R_3 \end{array} \left[\begin{array}{cc|c} 1 & 1 & -2 \\ 0 & 2 & -10 \\ 0 & 4 & k-4 \end{array}\right]$$

$$-2R_2 + R_3 \to R_3 \left[\begin{array}{cc|c} 1 & 1 & -2 \\ 0 & 2 & -10 \\ 0 & 0 & k+16 \end{array}\right]$$

From the bottom row of the last matrix, we can see that the stated vector equation will lead to a contradiction unless $k = -16$.

$\therefore$ $\begin{bmatrix} -2 \\ -12 \\ k \end{bmatrix}$ is in the span of $\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \right\}$ if and only if $k = -16$.

2. Using the techniques illustrated in Example 2.7 of your notes, balance the following chemical equation.

$$NH_3 + O_2 \to N_2 + H_2O$$

Let's define the molecules as column vectors thus:

$$\begin{bmatrix} \text{number of nitrogen atoms in one molecule} \\ \text{number of hydrogen atoms in one molecule} \\ \text{number of oxygen atoms in one molecule} \end{bmatrix}$$

The chemical equation is balanced by solutions to:

$$x_1 \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

This gives us the system of equations:

$$\begin{cases} x_1 \quad -2x_3 = 0 \\ 3x_1 \quad -2x_4 = 0 \\ 2x_2 \quad -x_4 = 0 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 3 & 0 & 0 & -2 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/3 & 0 \end{array}\right]$$

So the general solution to our balance equation is:

$$\begin{cases} x_1 = \frac{2}{3}x_4 \\ x_2 = \frac{1}{2}x_4 \\ x_3 = \frac{1}{3}x_4 \\ x_4 \text{ is free} \end{cases}$$

Since we can't have fractional parts of molecules, we let $x_4 = 6$.

$\therefore$ A balanced equation is:

$$4NH_3 + 3O_2 \to 2N_2 + 6H_2O$$