

MTH 261 Graded HW 1

Name

Key

This assignment is due at 6:00 PM on Wednesday, April 12

You may work on this assignment with your classmates or anybody else you please. You may get help from a tutor or even the instructor. What you may not do is simply copy somebody else's work – that completely obviates the purpose of the assignment. If you forget to complete the assignment before it is due, do not simply copy someone else's paper and turn that in ... that is not "working together," that is taking credit for somebody else's work. You should not be working on this in class right before it is due; you have five days to get this done – it should be completed well before ten minutes before it is due.

Problem 1

Consider the system
$$\begin{cases} 3x_1 + 2x_2 - 3x_3 - x_4 + 11x_5 = -1 \\ -2x_1 + x_2 + 2x_3 - 6x_5 = -5 \\ -x_1 - 4x_2 + x_3 - 3x_4 + 3x_5 = -13 \end{cases}$$
 . For this system:

- write down the associated augmented matrix as well as the reduced row echelon form of the matrix; find the reduced row echelon form on your calculator;
- write down the general solution to the system; do any necessary figuring on scratch paper;
- determine the specific solution where $x_3 = 3$ and $x_5 = -2$; show enough work so that your thought process is clear.

$$\left[\begin{array}{ccccc|c} 3 & 2 & -3 & -1 & 11 & -1 \\ -2 & 1 & 2 & 0 & -6 & -5 \\ -1 & -4 & 1 & -3 & 3 & -13 \end{array} \right] \sim \left[\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 3 & 12 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 & 5 \end{array} \right]$$

The general solution is:
$$\begin{cases} x_1 = x_3 - 3x_5 + 2 \\ x_2 = -1 \\ x_3 \text{ is free} \\ x_4 = 2x_5 + 5 \\ x_5 \text{ is free} \end{cases}$$

The specific solution when $x_3 = 3$ and $x_5 = -2$ is:

$$\begin{cases} x_1 = 11 \\ x_2 = -1 \\ x_3 = 3 \\ x_4 = 1 \\ x_5 = -2 \end{cases}$$

2. Manipulate $\begin{bmatrix} -2 & -4 & 3 & 10 & -2 \\ 1 & 2 & -2 & -7 & 3 \\ 3 & 6 & 3 & 15 & -7 \end{bmatrix}$ into reduced row echelon form **by hand** showing each step

in the process. Make sure that you annotate your work in a manner consistent with that shown in class. Please note that this matrix is not augmented – it does not represent a system of equations. This problem is merely about the process of manipulating a matrix into reduced row echelon form.

$$\begin{bmatrix} -2 & -4 & 3 & 10 & -2 \\ 1 & 2 & -2 & -7 & 3 \\ 3 & 6 & 3 & 15 & -7 \end{bmatrix} \quad R_1 \leftrightarrow R_2 \quad \begin{bmatrix} 1 & 2 & -2 & -7 & 3 \\ -2 & -4 & 3 & 10 & -2 \\ 3 & 6 & 3 & 15 & -7 \end{bmatrix}$$

$$\begin{array}{l} 2R_1 + R_2 \rightarrow R_2 \\ -3R_1 + R_3 \rightarrow R_3 \end{array} \quad \begin{bmatrix} 1 & 2 & -2 & -7 & 3 \\ 0 & 0 & -1 & -4 & 4 \\ 0 & 0 & 9 & 36 & -16 \end{bmatrix}$$

$$9R_2 + R_3 \rightarrow R_3 \quad \begin{bmatrix} 1 & 2 & -2 & -7 & 3 \\ 0 & 0 & -1 & -4 & 4 \\ 0 & 0 & 0 & 0 & 20 \end{bmatrix}$$

$$\begin{array}{l} -R_2 \rightarrow R_2 \\ \frac{1}{20}R_3 \rightarrow R_3 \end{array} \quad \begin{bmatrix} 1 & 2 & -2 & -7 & 3 \\ 0 & 0 & 1 & 4 & -4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} -3R_3 + R_1 \rightarrow R_1 \\ 4R_3 + R_2 \rightarrow R_2 \end{array} \quad \begin{bmatrix} 1 & 2 & -2 & -7 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$2R_2 + R_1 \rightarrow R_1 \quad \begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$