1. List the first 4 terms in the sequence of partial sums for the series $\sum_{k=1}^{\infty} k!$; show the calculations. (7 points)

2. List the first 4 terms in the sequence generated by $a_k = \begin{cases} 
2 & \text{if } k = 1 \\
3 & \text{if } k = 2 \\
6 \frac{a_{k-1}}{3} - \frac{a_{k-2}}{2} & \text{if } k > 2 
\end{cases}$. Show your calculations. (7 points)
3. Find the specific solution to the differential equation \( \frac{dy}{dx} = x^2 y + x y \) that passes through the point \((3,1)\). Make sure that you solve the solution equation for \( y \). (14 points)
4. A slope field for each of the following differential equations is shown in one of figures 1 - 6. For each equation, state the slope field (by figure number) that matches the equation. (3 points each)

\[ \frac{dy}{dx} = \ln(|y|) + \sin\left(\frac{\pi \sqrt{3}x}{3}\right) \] is shown in Figure ________.

\[ \frac{dy}{dx} = \ln(e^y) - \ln(|x|) \] is shown in Figure ________.

\[ \frac{dy}{dx} = \ln(|y|) - \cos\left(\frac{\pi \sqrt{3}y}{3}\right) \] is shown in Figure ________.
5. \[ \sum_{k=0}^{\infty} \frac{4}{5^k + 1} \] is a convergent geometric series; you do not need to prove this to me. What I need you to do is find the sum of the series; make sure that you show the calculation. (6 points)

6. \[ \sum_{k=1}^{\infty} \frac{3 \cdot (-4)^{-2k+3}}{5^{-3k-1}} \] is a divergent geometric series. This you do need to prove to me. Make sure that you show me all of the necessary algebra and that you state the basis for your conclusion. (9 points)
7. Consider the telescoping series \( \sum_{k=1}^{\infty} \left[ \frac{4}{k+4} - \frac{4}{k+6} \right] \). Establish a pattern for the sequence of partial sums and state the formula for this pattern. Then use that formula to determine and state the sum of the series. (14 points)
8. Consider the series \( \sum_{k=1}^{\infty} \frac{2^k}{k^2 3^k} \). Perform a Limit Comparison Test with the series whose term formula is \( b_k = \frac{1}{k^2} \) and state an appropriate conclusion. Please note that an appropriate conclusion might be "the test was inconclusive." (8 points)

9. \( \lim_{k \to \infty} \frac{\tan^{-1}\left(\frac{1}{k}\right)}{\frac{1}{k^2}} = \infty \). What, if anything, does this establish about the convergence or divergence of \( \sum_{k=1}^{\infty} \tan^{-1}\left(\frac{1}{k}\right) \)? Explain. (8 points)
10. For each series state whether the series is convergent or divergent and state a test you could perform to establish the convergence/divergence. Do not perform the test.

<table>
<thead>
<tr>
<th>Series</th>
<th>Con/Diverge?</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum_{k=1}^{\infty} \frac{1}{k}$</td>
<td>Diverge</td>
<td>Limit Comparison Test with the $p$-series $\sum_{k=1}^{\infty} \frac{1}{k}$</td>
</tr>
<tr>
<td>$\sum_{k=1}^{\infty} \frac{1}{k^2}$</td>
<td>Convergent</td>
<td>Comparison Test with the $p$-series $\sum_{k=1}^{\infty} \frac{1}{k^2}$</td>
</tr>
<tr>
<td>$\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$</td>
<td>Diverge</td>
<td>Comparison Test with the $p$-series $\sum_{k=1}^{\infty} \frac{1}{k}$</td>
</tr>
<tr>
<td>$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k} + 3}$</td>
<td>Convergent</td>
<td>Comparison Test with the $p$-series $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$</td>
</tr>
<tr>
<td>$\sum_{k=1}^{\infty} \frac{1}{2 + 3k}$</td>
<td>Diverge</td>
<td>Comparison Test with the $p$-series $\sum_{k=1}^{\infty} \frac{1}{3k}$</td>
</tr>
</tbody>
</table>

A correct answer for the first series has already been supplied to help you understand the directions (3 points each).